NONLINEAR, UNBONDED, POST-TENSIONED TENDON ELEMENT FOR CONCRETE STRUCTURES

By

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A general tendon element is developed to model the prestressing reinforcement for the analysis of either pretensioned or post-tensioned, prestressed concrete structures. Two versions of the element are provided. One is the tendon element without friction, which can be used for bonded prestressed concrete structures. The other is the tendon element with friction, which is mainly developed to simulate the external prestressing tendon used in post-tensioned, prestressed concrete structures.

The formulation of the tendon element·without friction is based on the continuity along the tendon. According to this criterion, the tendon stiffness matrix better models real structures. The initial force matrix is formed by using force eqilibrium at each node. The prestress loss due to the deformation of the structure is calculated from the nodal displacements. After the loss is calculated, the final tendon force is then evaluated.

The immediate losses, due to the friction and anchorage slip, are included in the tendon element with friction. After accounting for the losses, the initial force matrix is then formed. Because of the existence of friction at the harped points, the tendon element will be a combination of truss and continuous elements. The structural model is determined from the sliding condition at each node. Then, the formulation of the stiffness matrix can be determined from the structural model of the tendon element. A force distribution method is proposed to solve for the tendon force including the effects of friction. This method uses the unbalanced force matrix and the slip stiffness matrix in which the friction effects are considered.

Four examples are presented to verify the validity and applicability of the elements. The effects of friction on the strength of the structure are shown by a parametric analysis.

CHAPTER 1
INTRODUCTION

1.1 Background of External Prestressing

The development of segmental bridge construction, mostly pioneered in Europe, has been spreading into the United States and elsewhere in the world. The major advantages of this type of bridges are a reduced total cost and duration of construction, especially in medium and long span bridges [1,2,3].

Most of the segmental bridges use post-tensioned prestressing cables buried inside the concrete sections. They may either be bonded or unbonded. In such systems, the ducts of adjacent segments have to be carefully positioned in order to match precisely at all joints. Extra effort has to be made to place the ducts properly. In addition, the ducts interfere with casting and consolidation of concrete [4,5]. For economical reasons and quickened paced construction, external tendons have been adopted. In recent years, several bridges have been built with prestressing tendons placed outside the concrete. Due to the many advantages, an increased usage of external prestressing may be expected.

The concept of external prestressing was introduced by Dischinger [4]. He used them in the first two prestressed concrete bridges ever built between 1928 and 1936. In France and the United Kingdom, external prestressing has been used for the past several years. Jean Muller [4] was the first engineer in the United States to apply the concept of external prestressing to a series of bridges, such as Long Key, Seven Mile, Channel Five and Niles Channel bridges of the Florida Keys.

Not only the erection of new bridges but also the repair of existing bridges have contributed to the development of external prestressing techniques [4]. When existing bridges need additional prestressing and no empty , pre-buried duct is available, external prestressing is the only way to solve the problem. For new bridges, one of the advantages of external prestressing is the flexbility to replace the cables.

The design of externally prestressed bridges can be divided into three types [4]. The first is wholly external prestressing tendons which are continuous over several spans. The Long Key Bridge and the Seven Mile Bridge in Florida are good examples of this. They were constructed by using the span-by-span erection method. In this form of construction, the precast segments are assembled using post-tensioned cables. Prestressing tendons run though the inside of the box girder and are anchored in the pier segments. Some other bridges, such as Vallon des Fleurs, La Banquiere in France and Bubiyan in Kuwait, were built in a similar way [4]. The profile of external tendons used in Seven Mile bridge is shown in Fig. 1.1.

The second type is wholly external, non-continuous prestressing. An example is the Sermenaz Viaduct, built by Bouygues using the cantilever method. The cable pattern includes the classical cantilever and closure tendons which are placed outside the concrete and anchored in vertical ribs provided in each segment. According to Virlogeux [4], this type needs more anchorages and is less attractive than the other two groups.

The third one is mixed prestressing. The method was developed for the La Fleche bridge. Internal tendons, confined in the upper slab, were designed to resist the deadload only. After deck completion, external cables were used as continuous prestressing tendons and designed to balance all other loads [4].

Advantages of the external prestressing system have been disscussed by Virlogeux [4] Sowlat and Rabbat [5]. They can be explained as follows: First, duration of construction, a major economic concern for all construction companies, is reduced by eliminating duct laying and adjustment operations. Second, the layout of the tendon can be made more streamlined and the efficiency of the prestressing is thus improved. This is so because wobble friction loss does not exist and the total friction losses are reduced. Third, placing and consolidation of concrete

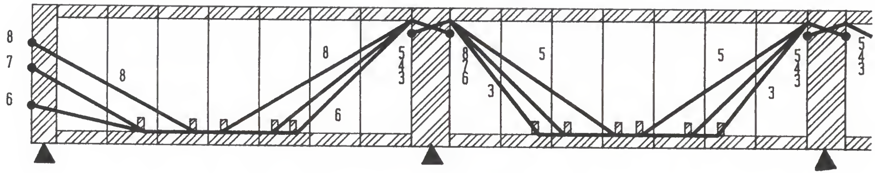


Fig. 1.1 External Prestressing in Seven Mile Bridge

are simplified and web thickness and the dead weight of the structure are reduced. Fourth, the injection procedure is simplied and the problem of concrete intrusion in the ducts during casting is eliminated. Therefore, it is easier to check the injection process and to access and repair inadequately grouted areas. Finally, replacement of broken and corroded tendons are possible with proper design.

1.2 Research Needs

Although external tendons have come into use gradually, very little information about testing and modeling is avaliable. Sowlat and Rabbat [5] tested the segmental concrete girders with external tendons and show that the ultimate strength is adequate under 1983 AASHTO Specifications [6] or ACI 318-83 Code [7].

For the computer analysis and design, there is a need to develop the model of the external tendon according to it's specific behavior. According to the AASHTO Specifications [6] or ACI 318-83 Code [7], external tendons belong to the category of unbonded tendons and the strength of the member is calculated based on these provisions. In reality, however, external tendons are not totally unbonded. They act semi-bonded at the deviation blocks(harping points) [8]. Although Soongswang [9] developed a model for external tendons assuming complete slip at each deviation block, further research about simulating the semi-bonded behavior of external tendons is still needed.

For unbonded tendons, the displacements along the tendon are not continuous because of lack of contact between the tendon and concrete. Also the stiffness of tendons are excluded in the analysis. But for external tendons, the tendon does come into contact with the concrete at the harping points, and then the deformation field is continuous. Therefore, the stiffness should be included.

1.3 Objectives of the Study and Scope of the Work

The objectives of developing the tendon element include the following:

First, provide a more reasonable and precise analysis for external prestressing. In general, the element can be applied to pre-tensioned bonded steels as well as post-tensioned unbonded steels.

Second, simulate the semi-bonded behavior at the deviation blocks and find out it's effects on the whole structure. The friction, induced by the deformation of the structure, is used to model the semi-bonded effects.

Third, provide the complete procedures for the analysis. This contains the tensioning process, with friction loss and anchorage slip, application of prestress load and external loads.

Fourth, study the friction effects on the ultimate strength of the structure. When the heavy load is applied on the structure, the friction at the deviation block becomes critical to the ultimate strength of the structure.

Fifth, study the effects of the tendon profile on the behavior of the structure. The tendon profile dominates the deformation and strength of the structure, and effects the magnitude of friction at the harping point. This friction can change the distribution of the tendon force and thus affects the behavior of the structure.

1.4 Arrangement of the Following Chapters

Chapter 2 presents the literature review about the computer modeling of the prestressing steel. The tendon element without friction, assuming complete slip at the harping point, is discussed in Chapter 3. Chapter 4 explains the tendon element with friction and presents a method to calculate the tendon force. Tensioning process, with friction loss and anchorage slip, are included in Chapter 5. Examples and numerical results are stated in Chapter 6. Conclusions and recommendations are included in Chapter 7.

Appendix A is the input guide for the tendon element. Appendices B and C provide the source program of the tendon element without and with friction, respectively.

CHAPTER 2
LITERATURE REVIEW

2.1 Introduction

The rapid development of external tendons has given rise to the need of precise analysis for segmental bridges. Computer models of prestressing steels have been given in previous works [9,10,11,12,13,14]. Truss elements or frame elements were used to model the prestressing steel in the analysis of prestressed concrete structures. The prestress losses induced by friction between prestressing steel and duct, elastic shortening, anchorage slip, shrinkage and creep of concrete and relaxation of the prestressing steel were considered in the analyses. The different behaviors between bonded and unbonded tendons were also included. From the past reseach, there is no specific element available to model external tendons. Although Soongswang [9] proposed a 2-D tendon element to simulate external presrtressing, the assumptions of the element can not really match the behavior of external tendon.

This chapter deals with the past research of prestressing steel, including application of prestress and the formation of stiffness. The following sections will discuss models of tendons, calculation of prestress losses, stiffnesses of tendons and internal tendon force.

2.2 Models of Tendons

Kang and Scordelis [10] divided a steel tendon into a series of straight segments for use in prestressed concrete frame analysis. These segments were modeled by truss elements or frame elements. The discrete representation of the prestressing steel or reinforcement, using one-dimensional elements, has been widely used [11]. The layout of tendon in the element is shown in Fig. 2.1. Greunen and Scordelis [12] and Millanes and Calzon [13] used a similar method to model prestressing steel in the analysis of prestressed concrete slabs and composite bridges, respectively. Soongswang [9] presented a tendon element, assuming complete slip at the harped point, to model external tendons in the analysis of segmental bridges. The profile and the degrees-of-freedom of the element is shown in Fig. 2.2.

The major difference between truss elements and tendon elements is in the formation of the stiffness for the tendons. The basic term of stiffness of a tendon can be expressed

$$k=E_pA_s/L \qquad (2.1)$$

in which

E_p = Young's modulus of the steel tendon.

A_s = Area of the tendon.

L = Length of the truss element or total length of the tendon.

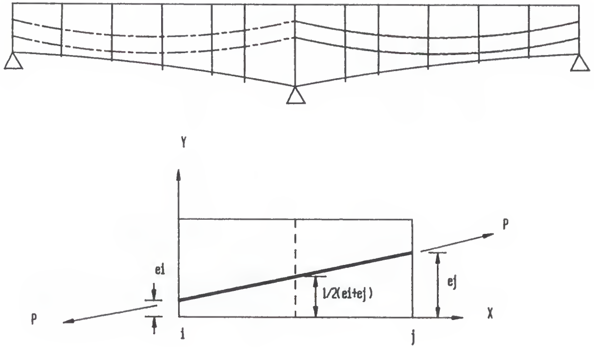


Fig. 2.1 Prestressing Steel Segment

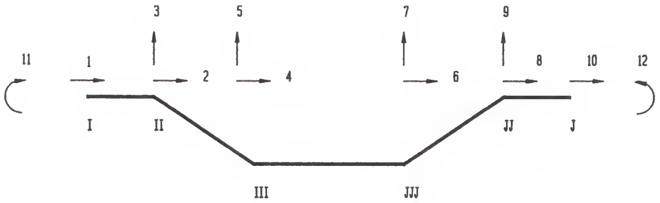


Fig. 2.2 Profile of the Tendon Element

The terms of the stiffness matrix of the tendon element are coupled to each other due to the continuity along the whole length. The movement of each degree-of-freedom of the tendon element will contribute the elongation of the tendon. But the movement of each node in the truss element's model only affects the elements connected to the node.

For the tendon with a smooth profile, such as parabola, the model of truss elements can be satisfied if enough elements are used. This is because the angle change of two neighbor elements is very small and can be ignored. But for a harped tendon, the model of truss elements mat not be reasonable. Because prestressing steel is basically a continuous member, the tension along the tendon should be the same excluding the friction effects. At the harped point the tension of each side will be the same, and the tendon is continuous. For the truss element model, the harped point is discontinuous. Finally, the different models will yield different structural behaviors, espacially if the stiffness of the tendon is taken into account in the analysis.

2.3 Calculation of Prestress Losses

In traditional and simplified analysis, only the key points of the span are concerned and the initial force along the tendon is assumed to be constant. In more precise analysis, the varied force along the tendon should be considered.

For pretensioning bonded tendons, losses of prestress are due to the instaneous and long-term deformations of the concrete. In this case, the initial force along the tendon is constant and equal to the jacking force. The losses are obtained internally as long as the change of elongation of the tendon has been evaluated [13].

For post-tensioned structures, losses of prestress are due to friction, anchorage slip, relaxation and deformations of the concrete [15,16]. These effects are described as follows:

2.3.1 Friction

When the tendon slides through the duct, friction resistance is developed. As a result, the tension will decrease as the distance or the angle change from the measured point to the jacking end increases [15,16]. The following equation is wellknown to calculate the friction loss

$$P_x = P_o\ e^{-(kx+\mu\alpha)} \tag{2.2}$$

where

P_x = Force at a distance x from the jack.

P_o = Force at the jack.

k = Wobble friction coefficient.

μ = Curvature friction coefficient.

α = Angle change in tendon profile over x.

The prestressing force at each of the steel segment ends can be calculated by using the above equation successively from the tensioning end. Kang and Scordelis [10] used the average of the values at the two ends as the segment force.

2.3.2 Anchorage Slip

This occurs when the tendon is anchored after the force is transfered to the concrete. Typically, the magnitude of slip is in the range from 1/8 in. to 1/4. in. depending on the prestressing system used [15].

To simplify the analysis, the slip is assumed to be uniformly distributed over the length of the tendon. Then the anchorage loss can be calculated from the expression [15]

$$\Delta f_{anc} = (\Delta L/L)E \qquad (2.3)$$

where

ΔL = amount of slip.

L = tendon length.

E_p = elastic modulus of the prestressing steel.

Actually, the loss is mostly confined to a region close to the jacking anchorage. For more elaborate analysis, an iterative procedure, accounting for friction, was used by Greunen and Scordelis [12]. The length affected by the slip loss, shown in Fig. 2.3, can be expressed by [17]

$$L_{anc} = 2\Delta LA_sE_p/P_o(1 - e^{-2qL_{anc}}) \qquad (2.4)$$

in which

$$q = (\mu\alpha+kx)/x \qquad (2.5)$$

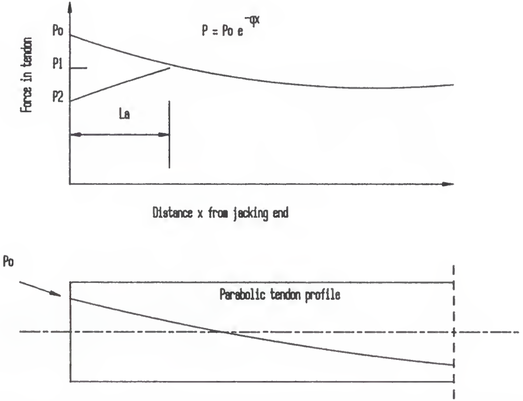


Fig. 2.3 Prestress Variation Before and After Anchorage

The above euation is based upon the assumption that the anchorage loss is develped over length L_{anc} and the average prestress along L_{anc} is equal to the average of P_o and P_2. After L_{anc} has been determined, the prestress at any point along the length L_{anc} can be evaluated by using Eq. (2.4).

2.3.3 Relaxation of Steel

When the prestressing steel has been held for some time, there is a gradual reduction of stress in the steel resulting from relaxation. The amount of relaxation includes the intensity of steel stress as well as time. The ratio of reduced stress f_p to initial stress f_{pi} can be estimated by [18]

$$f_p/f_{pi} = 1 - \log t /10(f_{pi}/f_{py} - 0.55) \qquad (2.6)$$

where

f_{py} = 0.1% offset yield stress.

t = the time in hours after stressing.

and f_{pi}/f_{fpy} is not less than 0.55.

2.3.4 Deformation of the Concrete

This deformation is caused by the stressing of tendons, application of external loads, the creep and shrinkage of concrete with time. The losses due to this deformation is automatically covered in the solution process [13].

2.4 Stiffnesses of Tendons

For pretensioned members, the stiffnesses of the tendons are used throughout the whole analysis. This is because the steel and the concrete are already bonded together when the prestress is released. This effect has been considered in past work [10,12,13,14].

For post-tensioned members, the prestress is transfered to the concrete gradually by a tensioning process. At force transfer, the stiffness of the prestressing steel is not included when the prestress and dead load are applied. After force transfer, the stiffness due to the prestressing steel is considered if the tendon is bonded. For unbonded tendons, there is no composite action between the steel and the concrete and the stiffness of the tendon is not included in the analysis. In this case, the unbonded tendons exert a stay-like action that contributes to the overall stiffness but they are not included in the stiffness of sections. Millanes and Calzon [13] used the following eqation to obtain the stiffness matrix

$$k = E_p A_p (Y_p)^T (Y_p)/L_o \tag{2.7}$$

Matrix (Y_p) has the form:

$$(Y_p) = (a_1, b_1, c_1, \ldots\ldots, a_n, b_n, c_n)_p \tag{2.8}$$

Directional coefficients a_i, b_i and c_i equal zero for all nodes not crossed by cable p and have the values for the remaining nodes.

Soongswang [9] formed the stiffness of the tendon element, based upon the continuous behavior along the whole length. But he did not include the stiffness through the whole analysis due to the unbonded behavior.

The above algorithm follows the classical method used in the prestressed concrete analysis [15]. With or without stiffness of the tendon corresponds to the net or transformed section used in the classical analysis.

2.5 Internal Tendon Force

Kang and Scordelis [10] used an iterative method to find the difference of the tendon length to calculate the tendon force. For bonded tendons, the strain increment is evaluated by

$$\Delta\epsilon = (L_c - L_p)/L_o \tag{2.9}$$

in which

L_c, L_p = the length of the prestressing steel segment for the current and previous iteration, respectively.

L_o = the original length based on the original locations of the two end points in the frame element.

For post-tensioned unbonded structures, the following eqation is used:

$$\Delta\epsilon = (\Sigma L_c - \Sigma L_p)/\Sigma L_o \tag{2.10}$$

where the summation is made for all the steel segments in the tendon. The total strain in current stage ϵ is obtained by summing up $\Delta\epsilon$. And the corresponding stress can be obtained from the stress-strain curve.

Usually the deformation field of the bonded tendons is continuous and the same as that of the concrete and the strain and tendon force can be calculated directly. But for the unbonded tendons, the deformation field is discontinuous everywhere except at the anchors and the tendon force can not be calculated directly.

Van Zyl and Scordelis [19] proposed the use of an average extension factor which makes it possible to directly calculate the change in force resulting from a change in tendon length for unbonded tendons. If a unit force is applied at both anchors, the tendon force in segment i, f_i, can be calculated by using Eq. (2.2) when the effect of friction is considered. The elongation in segment i due to the unit force is expressed by

$$\Delta L_i = l_i \; f_i/(E_p \; A_p) \qquad (2.11)$$

The total elongation, ΔL, of the tendon of n segments is

$$\Delta L = \Sigma \; \Delta L_i, \; i=1,2,3,\ldots,n \qquad (2.12)$$

The average extension factor is defined:

$$A_{ef} = (E_p \; A_p)/\Sigma(l_i \; f_i) \qquad (2.13)$$

Then the force change at the anchors is obtained:

$$\Delta P_o = \Delta L \; A_{ef} \qquad (2.14)$$

And the force change in each segment can be calculated by mutiplying $\Delta P_o \; f_i$.

Greunen and Scordelis [12] adopted the criterion for unbonded tendons and used the extension ratio in the analysis of prestressed concrete slab. The ratio is defined as the extension of a frictionless tendon over that of a tendon, including the effect of friction due to a unit force applied at both anchors, and is given by

$$R_e = L/\Sigma(l_i \; f_i) \geq 1 \; ; \; i=1,2,\ldots\ldots,n \qquad (2.15)$$

After calculating the total elongation, ΔL, and using the extension ratio, R_e, the change in anchor force, ΔP_o, is calcualted as

$$\Delta P_o = (A_p\ E_{sa}\ R_e)\Delta L/L \qquad (2.16)$$

where E_{sa} is the average modulus of elasticity of the prestressing steel.

CHAPTER 3
TENDON ELEMENT WITHOUT FRICTION

3.1 Introduction

The purpose of tendon element is to analyze the behavior of prestressed concrete structures which contain prestressing steel. Because of the different properties and behavior between concrete and steel tendons, it is more plausible to use a tendon element combined with concrete elements (shell elements) than to use only concrete elements in the analysis of prestressed concrete structures. Basically, the element can be used in pretensioning or post-tensioning tendons.

The tendon element introduced here has 4 or 6 nodes(Fig. 3.1) which connect to a shell or membrane element used to model concrete beams.

To be capable of handling the 3-D prestressing steel, the tendon element has twelve degrees of freedom, with two at node II and III and three at node JJJ and JJ and only one at node I and J, in the local coordinate system. In the global coordinate system, three degrees of freedom per node are required because of the 3-D capabilities.

The tendon element discussed here does not consider the force variation at the harping points, that is, sliding

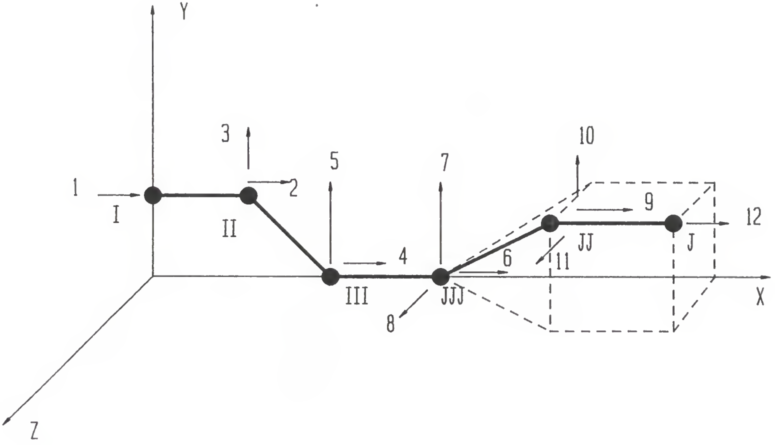


Fig. 3.1 Tendon Element in the Local Axes

will happen at all intermediate nodes. Therefore, the tendon element can be modeled as a continuous tension element. The tension at any point along the tendon is assumed to be the same. For external prestressing, more elaborate modeling, due to the force variation, will be discussed in the next chapter.

3.2 Formulation

3.2.1 Stiffness Matrix Without Friction

In general, the sag of the tendon compared to the span is very small, therefore the change of the angles is negligible. Small strain theory is applicable in this derivation. The derivation of the stiffness matrix without friction is developed in a local coordinate system q (Fig. 3.2).

The stiffness is derived by definition:

K_{ij}=the force at DOF i due to the unit displacement at DOF j. When $q_1=1$ and the other displacements are zero, then the deformation of the tendon is $\Delta l=-1$ and the force of the tendon will be equal to -EA/L. From the equilibrium of each node (Fig. 3.2), we can easily find the forces at each degree of freedom as

At node I

$K_{11}= EA/L$

At node II

$K_{21}= -EA/L(1-\cos\alpha_1)$

$K_{31}= -EA/L(\cos\beta_1)$

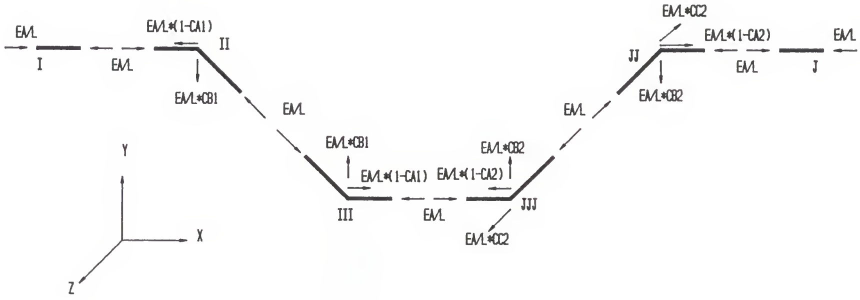


Fig. 3.2 Derivation of Stiffness Matrix (q=1)

At node III

$K_{41} = -K_{21}$

$K_{51} = -K_{31}$

At node JJJ

$K_{61} = -EA/L(1-\cos\alpha_2)$

$K_{71} = EA/L(\cos\beta_2)$

$K_{81} = EA/L(\cos\gamma_2)$

At node JJ

$K_{91} = -K_{61}$

$K_{10,1} = -K_{71}$

$K_{11,1} = -K_{81}$

At node J

$K_{12,1} = -EA/L$

where

E = modulus of the tendon.

A = area of the tendon.

L = total length of the tendon.

α_1, β_1 = the angles between segment II-III and X and Y axis, respectively.

α_2, β_2, γ_2 = the angles between segment JJJ-JJ and X and Y and Z axis, respectively.

Similarly, the other stiffnesses terms can be derived. The stiffness matrix can be stated as follows:

$$[K]_L = EA/L \left[\begin{array}{c|c|c|c|c|c|c|c|c|c|c|c}
q_1 & q_2 & q_3 & q_4 & q_5 & q_6 & q_7 & q_8 & q_9 & q_{10} & q_{11} & q_{12} \\
1 & -X11 & -Y1 & X11 & Y1 & -X12 & Y2 & Z2 & X12 & -Y2 & -Z2 & -1 \\
 & X11^2 & X11*Y1 & -X11^2 & -X11*Y1 & X11*X12 & -X11*Y2 & -X11*Z2 & -X11*X12 & X11*Y2 & X11*Z2 & X11 \\
 & & Y1^2 & -Y1*X11 & -Y1^2 & Y1*X12 & -Y1*Y2 & -Y1*Z2 & -Y1*X12 & Y1*Y2 & Y1*Z2 & Y1 \\
 & & & X11^2 & X11*Y1 & -X11*X12 & X11*Y2 & X11*Z2 & X11*X12 & -X11*Y2 & -X11*Z2 & -X11 \\
 & & & & Y1^2 & -Y1*X12 & Y1*Y2 & Y1*Z2 & Y1*X12 & -Y1*Y2 & -Y1*Z2 & -Y1 \\
 & & & & & X12^2 & -X12*Y2 & -X12*Z2 & -X12^2 & X12*Y2 & X12*Z2 & X12 \\
 & & & & & & Y2^2 & Y2*Z2 & X12*Y2 & -Y2^2 & -Y2*Z2 & -Y2 \\
 & \text{Symmetric} & & & & & & Z2^2 & X12*Z2 & -Y2*Z2 & -Z2^2 & -Z2 \\
 & & & & & & & & X12^2 & -X12*Y2 & -X12*Z2 & -X12 \\
 & & & & & & & & & Y2^2 & Y2*Z2 & Y2 \\
 & & & & & & & & & & Z2^2 & Z2 \\
 & & & & & & & & & & & 1
\end{array}\right] \tag{3.1}$$

where

$X1 = \cos\alpha_1$

$X2 = \cos\alpha_2$

$Y1 = \cos\beta_1$

$Y2 = \cos\beta_2$

$Z2 = \cos\gamma_2$

$X11 = 1 - \cos\alpha_1$

$X12 = 1 - \cos\alpha_2$

3.2.2 Transformation Between Local and Global Stiffnesses

Let [T] represent the transformation matrix between local and global displacements. We have

$$\{q\}=[T]\{r\} \tag{3.2}$$

where

$\{q\}$ is the matrix of local displacements.

$\{r\}$ is the matrix of global displacements.

In the same way, we can find

$$\{Q\}=[T]\{R\} \tag{3.3}$$

where

$\{Q\}$ is the force matrix in local coordinate.

$\{R\}$ is the force matrix in global coordinate.

The force-displacement relationship in local coordinate can be written

$$\{Q\}=[K]_L\{q\} \tag{3.4}$$

From Eq.(3.2)

$$\{Q\}=[K]_L[T]\{r\} \tag{3.5}$$

and, from Eq.(3.3),

$$[T]\{R\}=[K]_L[T]\{r\} \tag{3.6}$$

Using the property of orthogonality,

$$\{R\}=[T]^T[K]_L[T]\{r\} \tag{3.7}$$

Thus

$$\{R\}=[K]\{r\} \tag{3.8}$$

where

$$[K]=[T]^T[K]_L[T] \tag{3.9}$$

This is the equation most generally used for finding the member stiffness matrix in global coordinates [20].

3.2.3 Initial Force

For pretensioned members, the initial force, T, is the jacking force transfered to the concrete. For post-tensioned members, the initial force, T, is the force deducting all the instaneous losses from the jacking force. In this version, the force variation along the tendon is not considered and the initial force should be estimated before being used in the program.

For the harped tendon, the initial force is the balanced force on the nodes (Fig. 3.3). The direction of the initial force defined in ANSR [21] is the direction that the tendon is stressed. For example, when a straight tendon is being stressed in the simple supported beam, the direction of the initial force is shown in Fig. 3.4.

When the prestressing force, T, is applied to the tendon, the initial forces on each degree of freedom can be evaluated by the equilibrium equations at each node (Fig. 3.3). The local force matrix is expressed by

$$[F]_L = T \begin{bmatrix} -1 \\ X11 \\ Y1 \\ -X11 \\ -Y1 \\ X12 \\ -Y2 \\ -Z2 \\ -X12 \\ Y2 \\ Z2 \\ 1 \end{bmatrix} \tag{3.10}$$

T is the prestressing force. The other notations are the same as in the stiffness matrix. The force matrix in global coordinates, from Eq. (3.3), can be written

$$[F]=[T]^T[F]_L \tag{3.11}$$

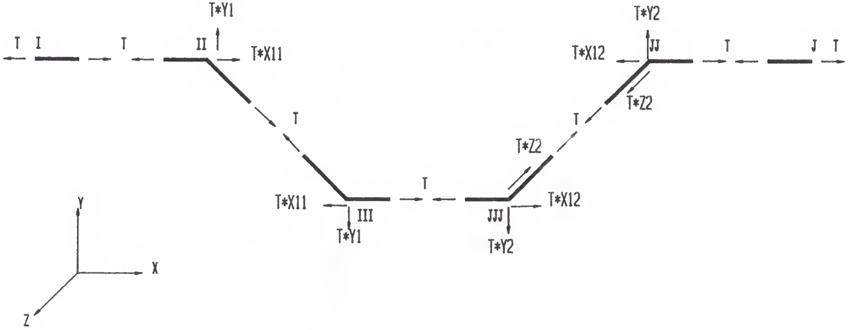


Fig. 3.3 Initial Force of Tendon Element

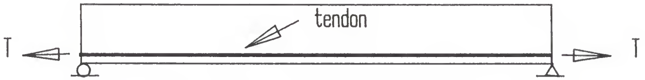


Fig. 3.4 Direction of the Initial Force

3.2.4 Resisting Internal Force

The resisting internal force is defined as the tendon force, including the losses due to the displacements, in the current step. Therefore, the magnitudes of resisting internal forces are the initial forces minus the forces induced by the nodal displacements. This can be calculated by sustituting T-ΔT into T in Eq. (3.10), where ΔT is the force due to the nodal displacements. To evaluate ΔT, we can use Eq. (3.8) to get the force matrix R in the global system. Because the tendon is continuous along the whole length, the force change will be constant. Only the force components at one node are needed for calculation. If node I exists, the components of the force change, ΔT, are R_1, R_2 and R_3. Otherwise, the components are R_4, R_5 and R_6. The sign convention used is that the resisting forces are the same sign as the initial forces.

3.3 Computer Implementation

ANSR [21] is used in this study. Eleven interface subroutines are called from the base program to perform element computations. Only six subroutines are utilized in tendon element for linear analysis without frictional forces. The functions of each subroutine are as follows:

3.3.1 Subroutine INEL11

This subroutine is called from the base program once for the tendon element. The purposes of the subroutine include the following:

First, input and initialize the element group data. The labeled common block /INFGR/ for the group of tendon elements is defined:

```
      COMMON /INFGR / NGR,NELS,MFST,IGRHED(10),NINFC,LSTAT,
     *                LSTF,LSTC,NDOF,DKO,DKT,EPROP(1000)
```

in which

NGR = Element group number.
= 11 for tendon element.

NELS = Number of elements within the group.

MFST = Element number of the first element in this group.

DKO = Initial stiffness damping factor.
= 0 for tendon element.

DKT = Current tangent stiffness damping factor.
= 0 for tendon element.

IGRHED(10)= Element group heading.

The above are read from the element input, and the others stated below are set up in this subroutine.

LSTAT = Length of words of state information variables.
= 108 for tendon element.

LSTF = Length of words of stiffness information variables.
= 648 for tendon element.

LSTC = Length of words of stiffness control variables.
= 0

NINFC = Length of words of variables in /INFEL/.

= 756 for tendon element.

NDOF = Number of element degrees of freedom.

= 18 for tendon element.

The element input data is read next and stored in the common block /INFEL/. The labled common block /INFEL/ for tendon element includes the following:

```
COMMON /INFEL/ IMEM,KST,LM(18),NODE(6),XX(6),YY(6),
               ZZ(6),PROP(1),AREA,XL,B(7),SEL,VTOT,
               STOT,TT(3,3),T,SKP(324)
```

in which IMEM, NODE(6), AREA and T are read from input, and the others are calculated in the program. The definitions of these variables are stated as follows:

IMEM = Element number.

KST = Stiffness update code.

= 1 (initialization).

= 0 (for linear case).

LM(18) = Location matrix.

NODE(6) = Node numbers from I to J.

XX(6)

YY(6) = Local coordinates of the six tendon

ZZ(6) nodes.

PROP = Young's modulus of the tendon.

AREA = Cross sectional area of the tendon.

XL = Element length.

B(7) = Direction parameters, see Eq. (3.13).

SEL = Current tendon force.
VTOT = Current tendon deformation.
STOT = Current tendon stress.
TT(3,3) = Transformation matrix.
T = Initial force of the tendon.
SKP(324) = Current stiffness matrix (=18*18).

The above, except SKP(324), are state information variables. SKP(324) is the stiffness information variable. The lengths of these variables are calculated and saved in COMMON /INFGR/.

The third step is to initialize the global degree of freedom numbers. This can be achieved by calling subroutine NCODLM eighteen times. Then the values are stored in LM array.

The fourth step is to transform the global coordinates of the nodes of the tendon into the local coordinates by calling subroutine TRAN1. In TRAN1 the transformation matrix TT is calculated by calling subroutine VECTOR and CROSS, and the local coordinates can be easily found by using the following formula:

```
XX(N)=TT(1,1)*X(N)+TT(1,2)*Y(N)+TT(1,3)*Z(N)
YY(N)=TT(2,1)*X(N)+TT(2,2)*Y(N)+TT(2,3)*Z(N)   (3.12)
ZZ(N)=TT(3,1)*X(N)+TT(3,2)*Y(N)+TT(3,3)*Z(N)
```

The subroutine saves the direction parameters in an array B. This is to avoid unnecessary recalculation.

Due to the geometry of the tendon element, it is easier to use direction parameters in order to find the components of the tendon force in the global axes. The direction parameters are defined:

$$\begin{aligned} B(1) &= \cos\beta_1 \\ B(2) &= \cos\beta_2 \\ B(3) &= \cos\alpha_1 \\ B(4) &= \cos\alpha_2 \\ B(5) &= \cos\gamma_2 \\ B(6) &= 1. - \cos\alpha_1 \\ B(7) &= 1. - \cos\alpha_2 \end{aligned} \qquad (3.13)$$

Finally, the stiffness matrix profile is calculated by calling the subroutine BAND and the element data is transferred to tape by calling the subroutine COMPAC.

3.3.2 Subroutine STIF11

This subroutine performs the calculation of the element stiffness. Because of excluding friction force in this stage, the subroutine is only called when the total element stiffness matrix is to be formed. The purposes of this subroutine are stated as follows:

First, compute the local stiffness matrix S (including the six-node tendon, five-node tendon and four-node tendon).

The stiffness matrix, derived in section 3.2.1, is calculated in this subroutine. For the general purposes, the stiffnesses of the four-node tendon and the five-node tendon are included here. These tendons are distinguished by the coordinates of the nodes, and their stiffnesses are modified from the six-node tendon.

Next, transform the local stiffness matrix S, twelve degrees of freedom, into the global stiffness matrix SK, eighteen degrees of freedom. The transformation, based upon Eq. (3.6), is calculated by calling subroutine TRAN2. The size of the transformation matrix is 12 x 18.

Finally, store the global stiffness matrix with the standard form (SKP) in the element common block INFEL.

3.3.3 Subroutine INIT11

This subroutine is called at the beginning of each load step. In original ANSR the prestressing force was not included in the options of initial forces, but changes have been made to adapt for this purpose. Then the subroutine init11 will be called by the base program. The purposes of the subroutine are as follows:

First, compute the initial forces, FI, using the prestressing force and the direction cosines. The local initial forces of the tendon, derived in section 3.2.2, are calculated by calling subroutine FORVER.

Finally, transform the local initial forces into the global forces RF. According to Eq. (3.11), the transformation, from twelve degrees of freedom to eighteen degrees of freedom, is done by calling subroutine TRAN3.

3.3.4 Subroutine RINT11

The subroutine performs the element force recovery. It is called in the beginning of the analysis and after each state determination. It's purposes include the following:

First, it evaluates the incremental tendon force, DSL, due to the displacements of the nodes. Because of neglecting frictional forces, the change of tendon force will be the same along the tendon. Therefore, only the forces at the end of the tendon are concerned. And the force components, SA, SB and SC, of the end node are calculated by using the global stiffness matrix, SKP, and global displacement matrix, q.

Second, it calculates the step loading factor, fac.

Next, the routine updates the total tendon force SEL and total stress STOT. The tendon force SEL can be easily found by substracting the tendon losses DSL from the previous tendon force which is the initial force in the linear case.

The routine then computes the resisting internal forces FT with respect to the local coordinates using the tendon force SEL. This is done by calling subroutine FORVER.

Finally, the local resisting internal forces, FT, are then transformed to global resisting internal forces, FE, by calling the subroutine TRAN3. There is no damping force in this element, so the force matrix FD is set to zero.

3.3.5 Subroutine STAT11

Subroutine STAT11 is the state determination subroutine. Since the element is linear, there is no state determination. Therefore, it sets the stiffness update code KST=0 for linear analysis without frictional forces. This is because the stiffness matrix is not changed during the analysis.

3.3.6 Subroutine OUTS11

The subroutine performs the output of the tendon element. It outputs element current state. If the print indicator KPR=0, no output is processed, otherwise, the output will include tendon force, stress and strain in the current state.

CHAPTER 4
TENDON ELEMENT WITH FRICTION

4.1 Introduction

The prestressing tendons used in this study, running inside the box girder (but not through the section), are completely unbonded from the concrete (Fig. 4.1). The locations of the tendons are controlled by the deviation blocks cast monolithically with segments [8,9]. Because of the high tension of the tendons, a high vertical force will be produced in the deviation block and consequently develop a large friction resisting force against the movement between the tendon and concrete. This friction and the bonding stress at the block yield the semi-bonded behavior. But the bonding stress is small compared to the friction and can be neglected or can be incorporated in the friction. When the bridge is deformed, the displacements of the deviation block will be accompanied by the deformation of the tendon. This deformation will induce changes in the tendon force and friction force and finally might change the structural behavior of the tendon.

Usually, friction losses in this type of unbonded tendons can be divided into two categories. One is the initial friction between tendon and duct when the tendons

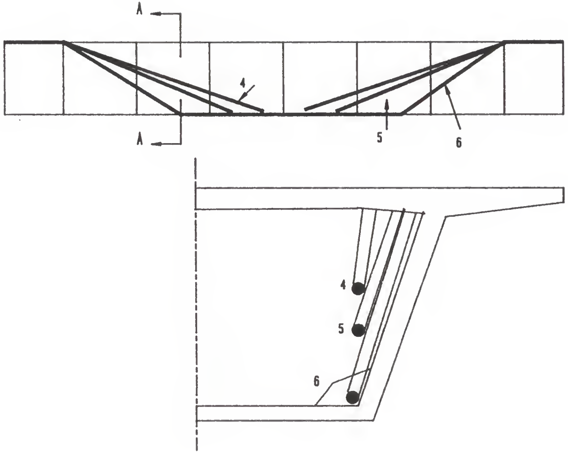


Fig. 4.1 Profile of the External Tendon

are being tensioned. The other is the friction between deviation block and tendon caused by the deformation of the stucture. The former is always taken into account in analyzing post-tensioned prestressed concrete structures. However, the latter is often ignored by most engineers. For bonded tendons this is reasonable, because sliding between tendon and concrete will not happen. But in external tendons, sliding between tendon and concrete block might happen. In this case, friction plays an important role and cannot be neglected. This chapter presents a rational method to determine the tendon force and find the effects of the friction on the behavior of the bridge.

4.2 Structural Model

There are twelve degrees of freedom for one tendon element in the local coordinate system (Fig. 3.1). Different from the frictionless tendon, the force along the tendon will not be constant but varied with the tendon segment. As in the finite element approach, the tendon is broken into five truss elements at the beginning of the solution process. After the displacements have been solved, the force of each segment can be found. When the difference of the ajacent segment forces is larger than the friction at the node, sliding will happen and the element will be treated as continuous. On the other hand, if friction is not overcome, the element will be discontinuous at the nodes and should be regarded as a truss element (Fig. 4.2).

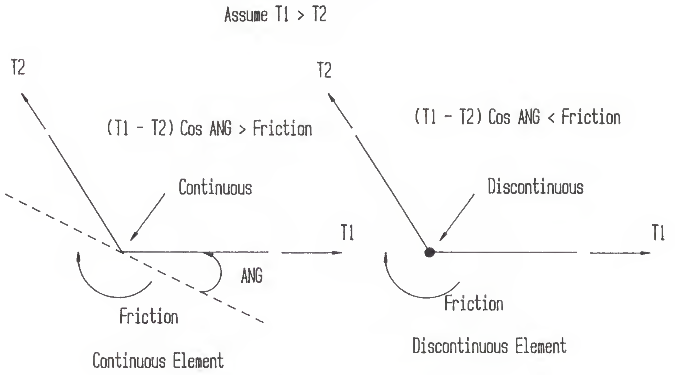


Fig. 4.2 Continuous and Discontinuous Elements

Therefore, the sliding conditions of the nodes strongly affect the structural behavior of the tendon.

After each node has been checked, the tendon will be a conbination of tendon elements and truss elements. There are many combinations of the tendon as shown in Fig. 4.3-4.5. When the areas of the tendon become large, the differences of structural behavior between tendon element and truss element will be obvious. In pratice, the external tendons are always used in heavily loaded bridges and the areas of tendons are not negligible. The stiffnesses of the tendons also form an important part of the structure. For these reasons the tendon element is used in analyzing the prestressed concrete bridge.

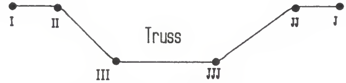
4.3 Formulation

4.3.1 Stiffness Matrix

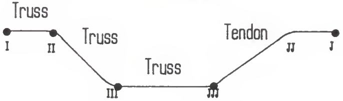
The stiffness matrix of the tendon element is dependent upon the structural model. As stated earlier, the element matrix is composed of a combination of tendon and truss stiffnesses. There are sixteen combinations of the structural model as shown in Fig. 4.2-4.4. To show the derivation of the stiffness matrix, consider case 7 composed of two truss elements and one tendon element.

The stiffness is derived by definition:

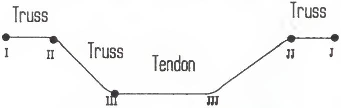
K_{ij}=the force at DOF i due to the unit displacement at DOF j. When q_2=1 and the other displacements are zero, then truss 1 will be in tension and the tendon will be in



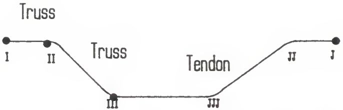
Case 1 : Truss Elements



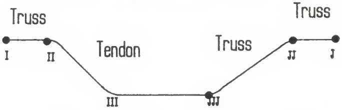
Case 2 : Combination of Truss and Tendon



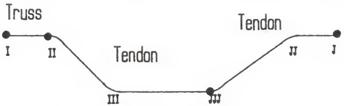
Case 3 : Combination of Truss and Tendon



Case 4 : Combination of Truss and Tendon

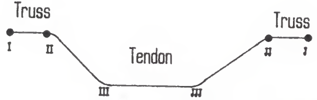


Case 5 : Combination of Truss and Tendon

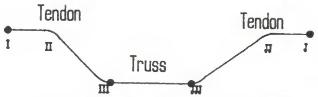


Case 6 : Combination of Truss and Tendon

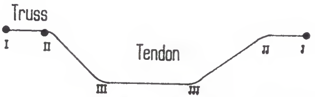
Fig. 4.3 Types of Structural Models (Part 1)



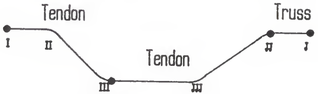
Case 7 : Combination of Truss and Tendon



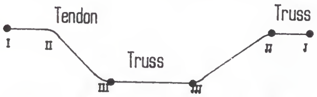
Case 10 : Combination of Truss and Tendon



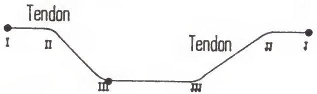
Case 8 : Combination of Truss and Tendon



Case 11 : Combination of Truss and Tendon

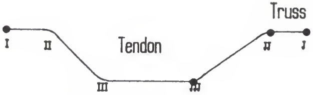


Case 9 : Combination of Truss and Tendon

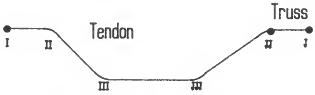


Case 12 : Two Tendons

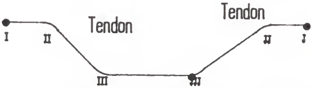
Fig. 4.4 Types of Structural Models (Part 2)



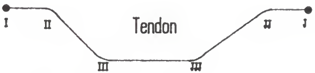
Case 13 : Combination of Truss and Tendon



Case 15 : Combination of Truss and Tendon



Case 14 : Two Tendons



Case 16 : Tendon

Fig. 4.5 Types of Structural Models (Part 3)

compression. The deformations of truss 1 and the tendon and truss 2 are 1 and $-\cos\alpha_1$ and 0, respectively. The force of truss 1 equals to EA/L_1. The force of the tendon will be uniform along the length and equals to $-EA/L$, where $L=L_2+L_3+L_4$. From the equilibrium of each node (Fig. 4.6), we can easily find the forces at each degree of freedom as

At node I

$K_{12}= -EA/L_1$

At node II

$K_{22}= EA(1/L_1+\cos\alpha_1{}^2/L)$

$K_{32}= -EA*\cos\alpha_1*\cos\beta_1/L$

At node III

$K_{42}= EA*\cos\alpha_1*(1 - \cos\alpha_1)/L$

$K_{52}= -K_{32}$

At node JJJ

$K_{62}= -EA*\cos\alpha_1*(1 -\cos\alpha_2)/L$

$K_{72}= EA*\cos\alpha_1*\cos\beta_2/L$

$K_{82}= EA*\cos\alpha_1*\cos\gamma_2/L$

At node JJ

$K_{92}= -EA*\cos\alpha_1*\cos\alpha_2/L$

$K_{10,2}= -K_{72}$

$K_{11,2}= -K_{82}$

At node J

$K_{12,2}= 0$

in which

α_1, β_1, γ_1 = the angles between segment II-III and X and Y axis, respectively.

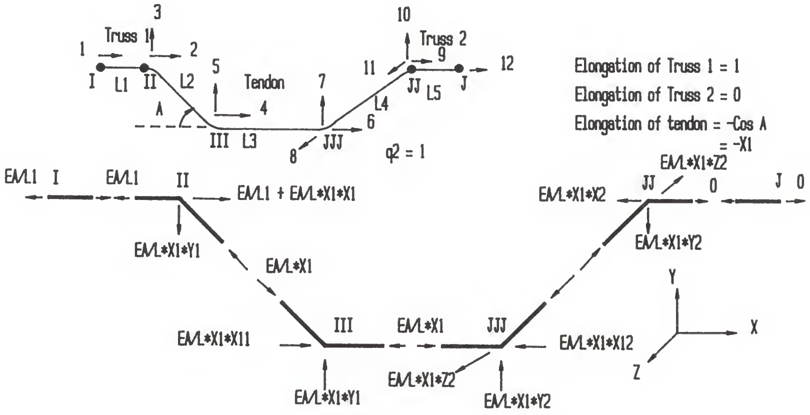


Fig. 4.6 Derivation of Stiffness Matrix (q2 =l)

α_2, β_2, γ_2 = the angles between segment JJJ-JJ and X, Y and Z axis, respectively.

Similarly, the other stiffnesses terms can be derived. The stiffness matrix can be stated as follows:

$$[K]_L = EA/L \left[\begin{array}{cccccccccccc}
q_1 & q_2 & q_3 & q_4 & q_5 & q_6 & q_7 & q_8 & q_9 & q_{10} & q_{11} & q_{12} \\
L/L_1 & -L/L_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 & L/L_1+X1^2 & -X1*Y1 & X1*X11 & X1*Y1 & -X1*X12 & X1*Y2 & X1*Z2 & -X1*X2 & -X1*Y2 & -X1*Z2 & 0 \\
 & & Y1^2 & -Y1*X11 & -Y1 & Y1*X12 & -Y1*Y2 & -Y1*Z2 & Y1*X2 & Y1*Y2 & Y1*Z2 & 0 \\
 & & & X11^2 & X11*Y1 & -X11*X12 & X11*Y2 & X11*Z2 & -X11*X2 & -X11*Y2 & -X11*Z2 & 0 \\
 & & & & Y1^2 & -Y1*X12 & Y1*Y2 & Y1*Z2 & -Y1*X2 & -Y1*Y2 & -Y1*Z2 & 0 \\
 & & & & & X12^2 & -X12*Y2 & -X12*Z2 & X12*X2 & X12*Y2 & X12*Z2 & 0 \\
 & & & & & & Y2^2 & Y2*Z2 & -Y2*X2 & -Y2^2 & -Y2*Z2 & 0 \\
 & \text{Symmetric} & & & & & & Z2^2 & -Z2*X2 & -Z2*Y2 & -Z2^2 & 0 \\
 & & & & & & & & X2^2+L/L_2 & X2*Y2 & X2*Z2 & -L/L_2 \\
 & & & & & & & & & Y2^2 & Y2*Z2 & 0 \\
 & & & & & & & & & & Z2^2 & 0 \\
 & & & & & & & & & & & L/L_2
\end{array}\right] \quad (4.1)$$

in which, the notations are defined in Chapter 3.

After the structure model has been determined, the number of sliding nodes can be evaluated and then the tendon stiffness can be formed and put in the corresponding block of the 12 x 12 element stiffness matrix. The truss stiffness then can be formed and added to the corresponding degrees of freedom in the element stiffness matrix. Whenever the structure model is changed, the element stiffness should be reformed.

4.3.2 Initial Force Matrix

The post-prestressing force is applied to the bridge in the form of equivalent loading. The derivation of the matrix was stated in Chapter 3 and omitted here. Fig. 3.3 shows the direction and magnitude of the initial force. The variation of the initial force along the tendon is not shown here and will be discussed in Chapter 5.

4.3.3 Resisting Internal Force Matrix

4.3.3.1 Determination of truss forces. After the displacements are solved, the deformation of each truss element can be calculated by using the formula

$$\Delta_i=[(dx_{i+1}-dx_i)^2+(dy_{i+1}-dy_i)^2+(dz_{i+1}-dz_i)^2]^{0.5} \qquad (4.2)$$

where

dx_i= x displacement at node i.

dy_i= y displacement at node i.

dz_i= z displacement at node i.

Then, the force of each truss element can be found:

$$S_i=TI_i+EA\Delta_i/L_i \qquad (4.3)$$

where

S_i= the force of truss element i.

TI_i= the initial force of the segment i.

Δ_i= displacement of truss element i.

L_i= length of truss element i.

4.3.3.2 Determination of the nodes' sliding conditions. If the force of each segment is known, then the friction at each node can be calculated. The friction model used assumes that the friction force is equal to the normal force times the coefficient of friction. At node i, if the driving force DF is larger than the static friction force SFF, then the node is sliding. Otherwise, the node will not slide. Because the tendon profile in the deviation block is not a strictly sharp angle but is a smooth curve. However, the transition lenth is very small, a circular curve can be assumed. Therefore, the direction of the normal force N is assumed to bisect the angle θ and the direction of the driving force DF is vertical to that of N as shown in Fig. 4.7. The magnitudes of the forces DF and SFF can be expressed as follows:

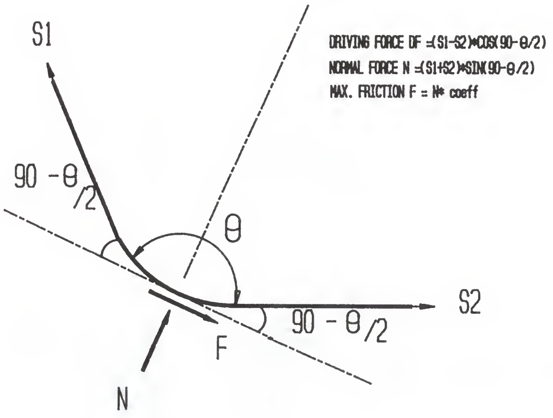
$$
\begin{aligned}
&DF=(S_{i+1}-S_i)\cos(90-\theta_i/2) \quad (S_{i+1} > S_i)\\
&N=(S_{i+1}+S_i)\sin(90-\theta_i/2) \qquad (4.4)\\
&SFF=\mu_s N
\end{aligned}
$$

where

S_{i+1}, S_i = the forces of the segments connected the node i.

μ_s= static friction coefficient betwwen tendon and concrete.

4.3.3.3 Determination of tendon forces. If both end nodes of the truss element do not slide according to the sliding criterion, the element will be a real truss element and the force S_i will be the member force and does not need to be revised. Otherwise, the tendon force will be determined by using force distribution based upon the stiffness of each



S1
DRIVING FORCE DF =(S1-S2)*COS(90-θ/2)
NORMAL FORCE N =(S1+S2)*SIN(90-θ/2)
MAX. FRICTION F = N* coeff
90 -θ/2
θ
S2
90 - θ/2
F
N

Fig. 4.7 Determination of Node's Sliding--One Node Case

truss. For expressing the concept, consider the case with one node's sliding which is shown in Fig. 4.7.

4.3.3.4 One node's sliding. Let S_i and S_{i+1} designate the force and of truss i and i+1, respectively. Then

$$S_i = TI_i + (AE\Delta_i)/L_i \qquad (4.5)$$

$$S_{i+1} = TI_i + (AE\Delta_{i+1})/L_{i+1}$$

where the notations were defined before. Assume S_i is larger than S_{i+1} and the driving force DF is larger than friction force SFF.

$$DF_i = (S_i - S_{i+1}) \cos(90 - \theta_i/2)$$

$$N_i = (S_i + S_{i+1}) \sin(90 - \theta_i/2)$$

$$SFF_i = N_i \mu_s \qquad (4.6)$$

$$DFF_i = N_i \mu_d$$

$$UBF_i = DF_i - DFF_i$$

where

θ_i= the angle formed by member i and i+1.

μ_d= dynamic friction coefficient between tendon and concrete.

DFF_i = the dynamic friction force at the node i.

UBF_i = the unbalanced force at the node i.

Then, distribute the unbanlanced force UBF_i to S_i and S_{i+1} by using the distribution factors KR_i and KR_{i+1}. The stiffness of each member is

$$K_i = AE/L_i \qquad (4.7)$$

$$K_{i+1} = AE/L_{i+1}$$

Since the friction will affect the force distribution, so the contribution of the friction is considered

$$K_f=\mu_d(K_{i+1}-K_i)\sin(90-\theta_i/2) \qquad (4.8)$$

Sum up the stiffnesses

$$\Sigma K=K_i+K_{i+1}+K_f \qquad (4.9)$$

Then, the distribution factors can be defined:

$$KR_i=K_i/\Sigma K \qquad (4.10)$$

$$KR_{i+1}=K_{i+1}/\Sigma K$$

Because S_i is larger than S_{i+1}, the friction force will reduce S_i and increase S_{i+1}. The final force of each segment can be calculated by

$$S_i=S_i - (UBF_i*KR_i)/\cos(90-\theta_i/2) \qquad (4.11)$$

$$S_{i+1}=S_{i+1} + (UBF_i*KR_{i+1})/\cos(90-\theta_i/2)$$

For the tendon without friction, the tendon force can be easily found by

$$S_i=S_{i+1}= TI_i + AE\Delta/L= T +AE\Delta/L \qquad (4.12)$$

where $\Delta=\Delta_i+\Delta_{i+1}$, $L=L_1+L_2+L_3+L_4+L_5$

By using Eq. (4.5-11) and pluging $\mu_s=\mu_d=0$, we can get the same result as the continuous tendon without friction. This shows that the force distribution can be applied for the tendon without friction as well.

4.3.3.5 Mutiple nodes' sliding. If more than one node is sliding, there will be more than one unbalanced force. Because of the existence of friction, there is difficulty in distributing the unbalanced forces with regard to direction. The unbalanced forces may be distributed to the nodes in opposite directions, thus it is hard to decide the final direction of the friction force. To solve the problem, a slip force-displacement relationship is formed.

4.3.3.6 Slip force-displacement relationship. The tendon is assumed as the truss structure at the beginning of the calculation of the tendon forces. If the nodes are sliding, then the nodes will be released. Suppose the tendon is modeled by the structure, with a roller support at each node, shown in Fig. 4.8. Since the element is continuous, the degree of freedom at each node is along the curve, connected with two adjacent segments. In this model, there is no external force but there is a fixed end force in each segment. The force-displacement relationship [20] can be stated as

$$\{R\} = [Kn]\,\{rl\} + \{SF\} \tag{4.13}$$

where

$\{R\}$ = the external force matrix.

= $\{0\}$

$[Kn]$ = the stiffness matrix of the truss structure.

$\{rl\}$ = the slip displacement matrix.

$\{SF\}$ = the fixed end force matrix.

If friction is considered, then the above is modified to

$$\{0\} = [Kn]\,\{rl\} + \{SF\} + \{F\} \tag{4.14}$$

where

$\{F\}$ = the friction force matrix between tendon and concrete. The friction force F includes the initial

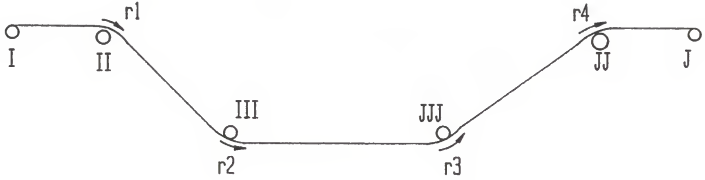


Fig. 4.8 Slip Degrees of Freedom in Tendon Element

friction FI, due to the fixed end force, and the incremental friction FD, due to the slip displacement (see Fig. 4.9). Break F into FI and FD and then rearrange Eq. (4.14), we can get

$$\{UBF\} = [Kn]\ \{rl\} + \{FD\} \tag{4.15}$$

where

$\{UBF\}$ = the unbalanecd force matrix.

$$= -\{SF\}-\{FI\} \tag{4.16}$$

Introduce the slip stiffness matrix [Kl] and Eq. (4.14) can be written

$$\{UBF\} = [Kl]\ \{rl\} \tag{4.17}$$

If SF_i is smaller than the maxmium initial friction FI_i at node i, then UBF_i will be equal to zero and the node won't slide. Therefore, the degrees of freedom of the above matrices are dependent upon the number of sliding nodes. At most there are four degrees of freedom in one tendon. The unbalanced force on degree of freedom i can be calculated by using Eq. (4.4-6).

4.3.3.7 Derivation of slip stiffness matrix. The slip stiffness matrix can be divided into two parts: One is based on the truss stiffness [Kn] and the other is the friction terms [Kf] induced by the distributive forces. Define

Kl_{ij} = the net force at DOF i due to the unit displacement at DOF j.

Kn_{ij} = the driving force at DOF i due to the unit displacement at DOF j.

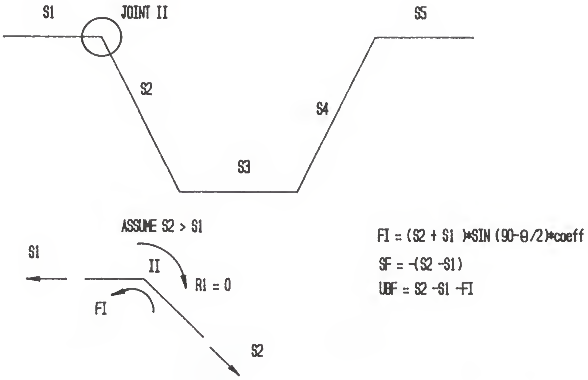


Fig. 4.9 Free Body Diagram of Joint II Before Force Distribution

Kf_{ij} = the friction force at DOF i due to the unit displacement at DOF j.

N_{ij} = the normal force at DOF i due to the unit displacement at DOF j.

Ki = the stiffness of each truss segment.

$= EA/L_i$

When $rl_1=1$ and the other displacements are zero, then the force of each segment will be

$$S_1=K1$$
$$S_2=-K2 \tag{4.18}$$
$$S_3=S_4=S_5=0$$

From the equlibrium of each node in tangential direction, we can find

$$Kn_{11}= K1 + K2$$
$$Kn_{21}=-K2$$
$$Kn_{31}=Kn_{41}= 0 \tag{4.19}$$

The nomal forces induced by the unit displacement are found from the equlibrium in normal direction

$$N_{11}= (K1-K2) \sin a$$
$$N_{21}= K2 \sin a \tag{4.20}$$
$$N_{31}= N_{41}= 0$$

where $a= 90 - \theta_1/2$

Next, the friction force at each node is determined by

$$Kf_{11}= \pm \mu_d N_{11}$$
$$= \pm \mu_d (K1-K2) \sin a$$

$$Kf_{21} = \mp \mu_d N_{21} \tag{4.21}$$

$$= \mp \mu_d K2 \sin a$$

$$Kf_{31} = Kf_{41} = 0$$

In matrix form, the above can be restated by

$$[Kf] = \mu_d [N] \tag{4.22}$$

The sign of Kf_{11} will be dependent upon the sign of UBF_1. This is because the direction of the incremental friction will be different from that of the initial friction when the unbalanced force is not compatible with the slip degree-of-freedom. If this happens the incremental displacement will cause the total normal force and the total friction to be reduced and finally the stiffness will be decreased. Similarly, the sign of Kf_{21} will be opposite to that of UBF_2. When $rl_1=1$, the segment II-III will be in compression and will decrease the total normal force and frictional force at node III. Therefore, the direction of the incremental friction will be opposite to that of the initial friction at node III. The derivation of slip stiffness matrix for $rl_1=1$ is shown in Fig. 4.10.

When $rl_2=1$ and the other displacements are zero, the force of each segment is given by

$$S_1 = 0$$

$$S_2 = K2 \tag{4.23}$$

$$S_3 = -K3$$

$$S_4 = S_5 = 0$$

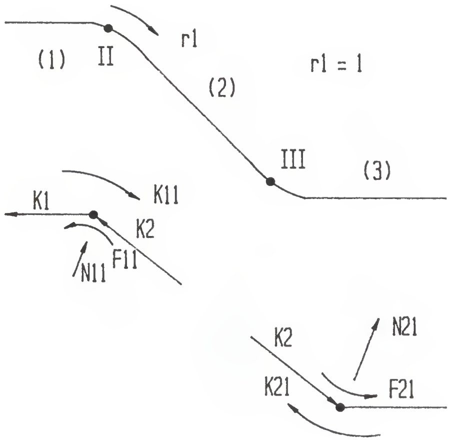


Fig. 4.10 Derivation of Slip Stiffness Matrix (r11 =1)

From the equlibrium of each node in both tangential and normal directions, the terms of $\{Kn\}_2$ and $\{N\}_2$ are given by

$$
\begin{aligned}
Kn_{12} &= -K2 \\
Kn_{22} &= K2+K3 \\
Kn_{32} &= -K3 \\
Kn_{42} &= 0
\end{aligned}
\tag{4.24}
$$

and

$$
\begin{aligned}
N_{12} &= K2 \sin a \\
N_{22} &= (K2-K3) \sin a \\
N_{32} &= K3 \sin b \\
N_{42} &= 0
\end{aligned}
\tag{4.25}
$$

The friction terms can be evaluated

$$
\begin{aligned}
Kf_{12} &= \pm \mu_d K2 \sin a \\
Kf_{22} &= \pm \mu_d (K2-K3) \sin a \\
Kf_{32} &= \mp \mu_d K3 \sin b \\
Kf_{42} &= 0
\end{aligned}
\tag{4.26}
$$

where $b = 90 - \theta_2/2$.

Again, the sign of Kf_{22} is the same as that of UBF_2 and the sign of Kf_{12} is compatible to that of UBF_1. But the sign of Kf_{32} is opposite to that of UBF_3 (Fig. 4.11). The reasons are as stated earlier. Similarly, the other stiffnesses can be derived. The slip stiffness matrix is expressed as follows:

$$[Kl] = [Kn] + [Kf] \tag{4.27}$$

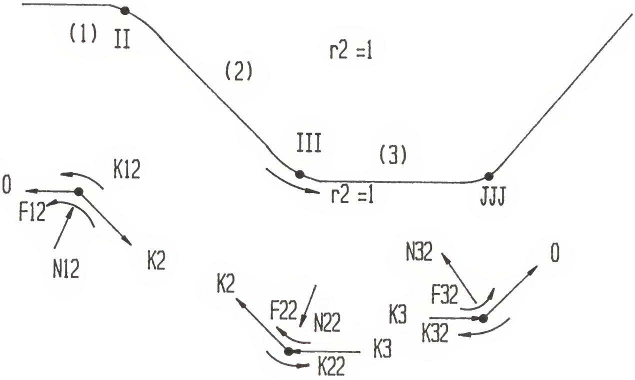


Fig. 4.11 Derivation of Slip Stiffness Matrix (r12 =1)

$$[Kn] = \begin{bmatrix} K1+K2 & -K2 & 0 & 0 \\ -K2 & K2+K3 & -K3 & 0 \\ 0 & -K3 & K3+K4 & -K4 \\ 0 & 0 & -K4 & K4+K5 \end{bmatrix} \qquad (4.28)$$

$$[Kf]= \mu_d \begin{bmatrix} \pm(K1-K2)\sin a & \pm K2 \sin a & 0 & 0 \\ \mp K2 \sin a & \pm(K2-K3)\sin a & \pm K3 \sin a & 0 \\ 0 & \mp K3 \sin b & \pm(K3-K4)\sin b & \pm K4 \sin b \\ 0 & 0 & \mp K4 \sin b & \pm(K4-K5)\sin b \end{bmatrix} \qquad (4.29)$$

We have to notice that the [Kl] matrix is not symmetrical if the profile of the tendon is not symmetrical or the loading is not symmetrical.

4.3.3.8 Determination of internal forces. After the unbalanced force matrix and the slip stiffness matrix are formed, the slip displacement {rl} can be solved by using

$$\{rl\}= [Kl]^{-1} \{UBF\} \qquad (4.30)$$

The internal force of each segment is dependent upon the relative slip displacement in each segment. Therefore, the force of each segment can be expressed by

$$\begin{aligned} Q_1 &= S_1 + K1\, rl_1 \\ Q_2 &= S_2 + K2(rl_2 - rl_1) \\ Q_3 &= S_3 + K3(rl_3 - rl_2) \qquad (4.31) \\ Q_4 &= S_4 + K4(r_4 - r_3) \\ Q_5 &= S_5 - K5\, rl_4 \end{aligned}$$

4.4 Solution Strategy

The determination of the tendon forces will be strongly related to the sliding conditions of the nodes. Not only one node can slide but at most four nodes can slide. A comprehensive method is proposed to solve the problem. The procedures can be stated as follows:

1. Assume the tendon is a combination of five truss elements and apply the initial forces to the structure.
2. Find the displacements at the nodes and the force of each truss element.
3. Find the sliding condition of each node by using Eq. 4.4.
4. Decide the structural model of the tendon.
5. Solve the tendon forces by using the method stated earlier and calculate the resisting internal forces.
6. Reform the stiffness matrix.
7. Resolve the structure.
8. Repeat procedure 2-7 until the forces are balanced.

4.5 Computer Implementation

4.5.1 Introduction

The tendon element with friction is installed in ANSR. Six interface subroutines are utilized for this element. The element is modified from the frictionless tendon element which is included in Chapter 3. The programming maintains the fundamental frame of the original version except the necessary changes for the specific purpose. In subroutine INEL, INIT and OUTS only a few portions are changed or added. But in subroutine STIF, STAT and RINT nearly all the programming is rewritten because the approach is totally different. The source code for these subroutines are contained in Appendix C. The functions of each subroutine are stated as follows:

4.5.2 Subroutine INEL11

The subroutine is called first when the elment is being used. The purposes of the subroutine include the following:

First, input and initialize the element group data and store the informations in labeled common block /INFGR/. The variables are defined:

```
 COMMON /INFGR/ NGR,NELS,MFST,IGRHED(10),NINFC,LSTAT,
                  LSTF,LSTC,NDOF,DKO,DKT,EPROP(4,250)
```

in which, the following variables are changed:

LSTAT = 170

LSTF = 648

NINFC = 818

The others are the same as those described in Chapter 3.

Next, read the element data and store the informations in the common block /INFEL/. The labeled common block /INFEL/ for this element includes the following:

```
 COMMON /INFEL/ IMEM,KST,LM(18),NODE(6),INS(6),INSO(6),XX(6),
               YY(6),ZZ(6),PROP(3),AREA,XL(5),B(7),SEL(5),
               STOT(5),TT(3,3),T,QL(12),SKP(324)
```

in which IMEM, NODE(6), AREA and T are read from input, and the others are calculated in the program. The definitions of new variables are stated as follows:

INS(6) = Code of node's sliding.
= 0 for no sliding.
= 1 for sliding.
= 0 for initialization.
INSO(6) = Previous code of node's sliding.
= 0 for initialization.
PROP(1) = Young's modulus of the tendon.
PROP(2) = Static friction coefficient between tendon and concrete.
PROP(3) = Dynamic friction coefficient between tendon and concrete.
XL(5) = Length of each segment in the tendon.
SEL(5) = Current tendon force of each segment.
STOT(5) = Current tendon stress of each segment.
QL(12) = Current displacements in local coordinate system.

The third step is to initialize the global degree of freedom numbers. This is done by calling subroutine NCODLM eighteen times. Then the values are stored in LM array.

The fourth step is to transform the global coordinates of the nodes of the tendon into the local coordinates by calling subroutine TRAN1. And then store the local coordinates in array XX(6), YY(6) and ZZ(6). The transformation is discussed in Chapter 3.

The fifth step is to evaluate the direction parameters and store in an array B.

The next step is to initialize array INS(6), INSO(6) and QL(12). From the solution strategy, the element is assumed to be a combination of five truss elements at the beginning. Therefore, the array INS and INSO are set to zero in this stage. Finally, the stiffness matrix profile is determined by calling the subroutine BAND and the element data is transferred to tape by calling the subroutine COMPAC.

4.5.3 Subroutine STIF11

This subroutine is used to form the element stiffness. Because of friction, the stiffness of the tendon will not be unique, however, will be strongly dependent upon the nodes' sliding conditions during the iterative analysis. The nodes' sliding conditions dominate the structural model, and furthermore affect the element stiffness. Therefore, the structural model should be determined before the stiffness matrix is formed. The functions of this subroutine include the following:

First, initialize the local stiffness matrix ST(12x12).

Second, compute the matrix ST according to the nodes' sliding conditions. There are sixteen combinations for the structural model. The flow chart for deciding the structural model and the stiffness are shown in Fig. 4.12. The element stiffness is actually a combination of tendon stiffness and

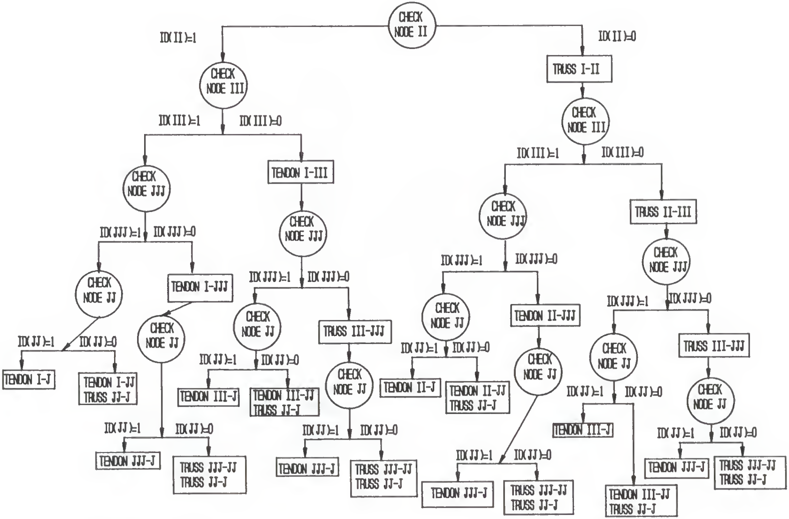


Fig. 4.12 Flow Chart for Determination of Structural Model

truss stiffness. The formation of tendon stiffness was provided in Chapter 3 and will not be stated here. Also, the derivation of combination of tendon and truss stiffness was stated earlier. There are several subroutines set up seperately in this program in order to provide the different element stiffness for the specific structural model. These include subroutine TRU46, TEN46, TT36, TEN36 and TEN16. The purpose of subroutine TRU46 is to form the stiffness for the truss structure from node JJJ to node J. Subroutine TEN46 is provided to form the stiffness for the tendon element from node JJJ to node J. Subroutine TT36 is used for the combination of the tendon element, from node III to node JJ, and the truss element, from node JJ to node J. Subroutine TEN36 is used for the tendon element from node III to node J. And subroutine TEN16 is used for the tendon elewments, from node I to node J, or from node I to node JJ, or from node II to node JJ, or from node II to J.

Next, transform the local stiffness matrix ST, twelve degrees of freedom, into the global stiffness matrix SK, eighteen degrees of freedom. The transformation, identical to the one dicussed before, is done by calling subroutine TRAN2. The 12 x 18 transformation matrix is used in this calculation.

Finally, store the global stiffness matrix with the standard form (SKP) in the element common block INFEL.

4.5.4 Subroutine INIT11

The main purpose of this subroutine is to apply the prestressing forces to the structure. It is called at the beginning of each load step. The functions of this subroutine are expressed as follows:

The first step is to calculate the loading factor fac which is the fraction of the initial load to be included in the current load increment. Next, multiply the factor fac to initial force T to get current initial force matrix P and determine the equvalent loading by calling subroutine FORVER. Subroutine FORVER has been changed to adapt the friction version. Because the tendon force might not be the same along the tendon due to friction. Therefore, P matrix was used instead of a constant value. The local force matrix is expressed by

$$[F]_L = \begin{bmatrix} -P(1) \\ P(1) - P(2)*B(3) \\ P(2)*B(1) \\ P(2)*B(3) - P(3) \\ -P(2)*B(1) \\ P(3) - P(4)*B(4) \\ -P(4)*B(2) \\ -P(4)*B(5) \\ P(4)*B(4) - P(5) \\ P(4)*B(2) \\ P(4)*B(5) \\ P(5) \end{bmatrix} \qquad (4.32)$$

in which B matrix was defined earlier.

The final step is to transform the local force matrix into the global force matrix by calling subroutine TRAN3. The transformation matrix was stated earlier and the formulation was based upon Eq. (3.11) in Chapter 3.

4.5.5 Subroutine STAT11

The subroutine provides current state information which dominates the change of the element stiffness. This can be done by comparing the current nodes' sliding conditions with the previous nodes' sliding conditions. If there are no changes in sliding conditions then the state code KST will be set to zero, otherwise KST will be set to one. But before the comparisons, the new silding conditions should be determined first. The procedures for determination of sliding conditions basically follows the solution strategy discussed earlier. It's purposes include the following:

First, it sets the previous sliding code INSO to the current code INS before recalculates the current code.

Second, it transforms the global nodal displacements Q into local nodal displacements and updates the current local nodal displacements QL. The transformation, based upon Eq. (3.2), is done by calling subroutine TRAN4.

Third, it calcualtes the additional deformation TD (not including the deformation due to initial force) of each truss element using the relative nodal displacements.

Fourth, it determines the loading factor fac used in the calculation of the total resisting internal forces. Fac is the total fraction of the initial load that has been applied.

Fifth, it initializes the force of each tendon segment SEL which is set to the corresponding truss force TRUSF. The truss force is calculated by adding the initial force T to the nodal force induced by the additional deformation TD. The step matches the assumption that the tendon is supposed to be five truss elements at the beginning of the solution process.

Sixth, it initializes the current sliding index array INS.

Seventh, it determines the sliding condition of each node by comparing the driving force DF with the maximum friction force FF. If the node slides, calculates the unbalnced force. Otherwise, sets the unbalanced force to zero. Then the structural model can be decided according to the nodes' sliding conditions.

Eighth, it forms the slip stiffness matrix SF and solves the slip displacements R. The slip stiffness matrix SF includes the non-frictional part AK, independent of the direction of the unbalanced force, and the frictional part BK, dependent upon the direction of the unbalanced force. The slip displacements are solved by calling subroutine SOL. The major task in subroutine SOL is to determine the slip degrees of freedom of the tendon first and then retrieve the corresponding slip stiffness matrix and the unbalanced force matrix and finally call equation solver subroutine CROUT to get displacements.

Ninth, it calculates the tendon force according to Eq. (4.31). Next, recheck the unsliding nodes by comparing the updated sliding conditions INS and the sliding conditions of the last iteration IN. If the two sliding indices are different, then repeat the procedures from seventh to ninth until they are the same.

Finally, set state code KST by comparing the sliding indices INS and INO.

4.5.6 Subroutine RINT11

The subroutine evaluates the element force recovery. It is called in the beginning of the analysis and after each state determination. Because the force of each segment SEL is already determined from subroutine STAT11, the force does not need to be calculated here. The purposes of this subroutine are

The first step is to calculate stress STOT of each segment of the element.

Next step is to form the resisting internal force matrix FT by calling subroutine FORVER. And then, transform FT in local axes into FE in global axes by calling subroutine TRAN3.

The final step is to set damping force matrix FD. The damping force is not considered in the tendon element, so the force matrix is set to zero.

CHAPTER 5
TENSIONING PROCESS

5.1 Introduction

For the pretensioned member, the initial force along the whole length can be regarded as a constant. But for the post-tensioned member, the initial force will not be constant because of friction and anchorage slip. The instaneous losses should be accounted for in the initial force.

Friction loss is developed between the steel and the duct when the steel slides through the duct. The total friction loss is the sum of the wobble friction due to the unintentional misalignment, and the curvature friction due to the intentional curvature of the tendon [15]. For external prestressing, the wobble friction does not exist because the tendon is outside of the concrete and no reaction is provided for the friction. Therefore, only the curvature friction is developed in the external tendons. Tensioning at one end is most commonly used for external prestressing but tensioned at both two ends is also included in this chapter.

Anchorage slip will happen in pretensioned members as well as post-tensioned members. But slip loss is insignificant in beams, pretensioned by long-line method, due to the great length of the tendon and often is ignored. The magnitude depends upon the particular prestressing system or hardware used and is available from the manufacturer or from laboratory tests. Usually, the loss is more important in short tendons.

The following sections will provide the method to calculate the immediate losses.

5.2 Friction Loss During Tensioning

5.2.1 Tensioning at One End

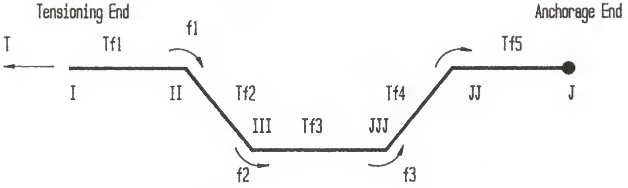
If the tendon is stretched at one end and anchored at the other end, the tendon force will gradually decrease with the distance from the point to the tensioned end. The curvature friction loss can be predicted by using Eq. (2.2). However, the direction of the friction, described in Chapter 4, is assumed to bisect the angle between two segments. According to this assumption, the equation can be simplified.

Suppose the tensioned end is at node I, the tendon force of each segment, shown in Fig. 5.1, is expressed by T_{f1}, T_{f2}, T_{f3}, T_{f4} and T_{f5}. Let f_1, f_2, f_3 and f_4 designate the friction at node II, III, JJJ and JJ, respectively. If the jacking force is T, then

$$T_{f1} = T$$

$$T_{f2} = T - f_1$$

$$T_{f3} = T - f_1 - f_2 \qquad (5.1)$$



Tf1 = T

Tf2 = T -f1

Tf3 = T - f1 - f2

Tf4 = T - f1 - f2 - f3

Tf5 = T - f1 - f2 - f3 - f4

Fig. 5.1 Friction Loss During Tensioning at One End

$$T_{f4} = T - f_1 - f_2 - f_3$$

$$T_{f5} = T - f_1 - f_2 - f_3 - f_4$$

The friction loss at each node is derived by the following: At node II, the normal force, N, is (shown in Fig. 5.2)

$$N = (T_{f1}+T_{f2})\sin a$$

$$= (2T - f_1)\sin a \tag{5.2}$$

where $a = (90-\theta_1/2)$

From the classical Coulomb's friction law, the friction, f_1, equals to the normal force, N, times the curvature friction coefficient, μ_{di}.

$$f_1 = \mu_{di}\ N \tag{5.3}$$

Substitute N, from Eq. (5.2), into Eq. (5.3)

$$f_1 = \mu_{di}\ (2T - f_1)\sin a \tag{5.4}$$

Rearrange the above equation, put f_1 on the same side We get

$$f_1(1+\mu_{di}\sin a) = 2\mu_{di}T\sin a \tag{5.5}$$

Solve f_1

$$f_1 = 2\mu_{di}T\sin a/A \tag{5.6}$$

where $A = 1 + \mu_{di}\sin a$

Similarly, the friction at node III, JJJ and JJ can be expressed by

$$f_2 = 2\mu_{di}(T-f_1)\sin a/A \tag{5.7}$$

$$f_3 = 2\mu_{di}(T-f_1-f_2)\sin b/B \tag{5.8}$$

$$f_4 = 2\mu_{di}(T-f_1-f_2-f_3)\sin b/B \tag{5.9}$$

where $b = 90 - \theta_2/2$ and $B = 1 + \mu_{di}\sin b$

After determiming the friction loss at each node, the tendon force can be calculated by using Eq. (5.1).

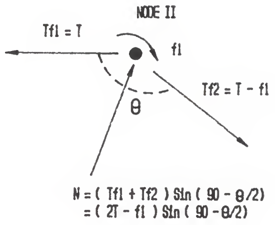

NODE II
Tf1 = T
f1
Tf2 = T - f1
θ
N = (Tf1 + Tf2) Sin (90 - θ/2)
= (2T - f1) Sin (90 - θ/2)

Fig. 5.2 Derivation of Friction Loss at Node II

5.2.2 Tensioning at Both Ends

If the tendon profile is symmetrical and stretched at both ends, the tendon force T_{f5} and t_{f4} will equal to T_{f1} and T_{f2}, respectively. And Eq. (5.1) can be adjusted to the following:

$$T_{f1} = T_{f5} = T$$

$$T_{f2} = T_{f4} = T - f_{f1} \qquad (5.10)$$

$$T_{f3} = T - f_{f1} - f_2$$

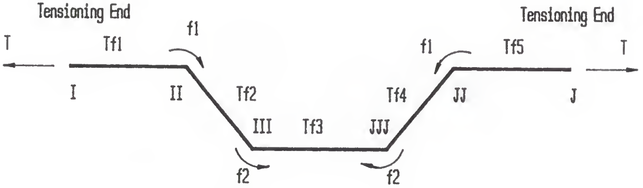
The magnitude of f_1 and f_2 are described in Eq. (5.6) and Eq. (5.7). Fig. 5.3 shows the tendon force with tensioning at both ends.

5.3 Anchorage Slip

5.3.1 Tensioning at One End

For external prestressing, Eq. (2.3-4) can not be applied because of the high friction at the harping point and no wobble friction. The slip loss will be resisted by the reverse friction at each node. The length affected by the loss will depend upon the reverse friction at each node. If the friction is high enough at the particular node, the loss will be stopped at that node. Otherwise, the effect will extend to the next node and may affect the whole tendon.

Let T_{s1}, T_{s2}, T_{s3}, T_{s4} and T_{s5} be the tendon force after the anchorage loss. Given the length of slip loss ΔL, the loss of the tendon force at segment I-II, Δf_{anc}, can be expressed by



$Tf1 = Tf5 = T$

$Tf2 = Tf4 = T - f1$

$Tf3 = T - f1 - f2$

Fig. 5.3 Friction Loss During Tensioning at Both Ends

$$\Delta f_{anc} = \Delta L\ E_p/L_1 \tag{5.11}$$

where L_1 is length of the segment I-II.

The tendon force of segment I-II is then

$$T_{s1} = T_{f1} - \Delta f_{anc} \tag{5.12}$$

The procedures to determine the intial force after slip loss are explained as follows (shown in Fig. 5.4):

5.3.1.1 Determine the sliding condition at node II. According to the sliding criterion described in Chapter 4, compare T_{s1} and T_{f2} and determine the sliding condition. If the node II does not slide, the tendon force will remain the same and no adjustment is necessary. On the contrary, if node II slides, then the tendon forces of segment I-II and II-III should be updated.

The procedures are similar to those described in Chapter 4 except for the slip stiffness matrix. The truss stiffness [Kn] stays the same but the signs in the friction matrix [Kf] will be somewhat different, because the effect of the slip loss will decrease with the distance from the tensioning end. Therefore, the direction of the unbalanced force in this case will be fixed, the sign in the matrix [Kf] will be constant. To distinguish the difference, the friction matrix, $[Kf]_i$, is restated

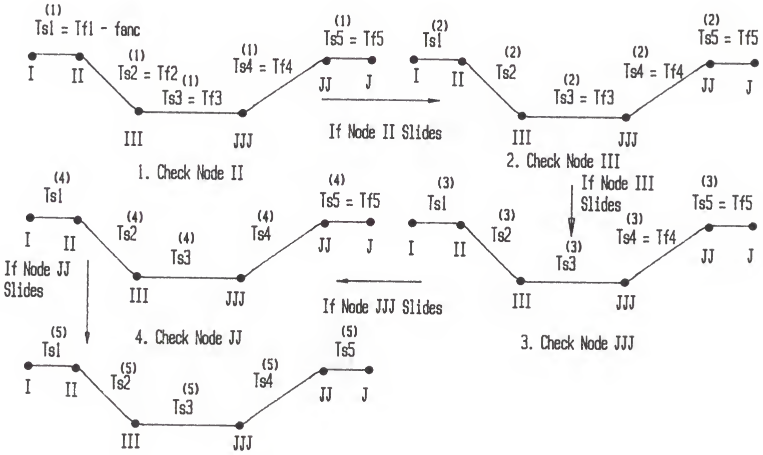


Fig. 5.4 Anchorage Slip Loss (Tensioning at One End)

$$[Kf]_i = \mu_{di} \begin{bmatrix} (K1-K2)\sin a & K2 \sin a & 0 & 0 \\ -K2 \sin a & (K2-K3)\sin a & K3 \sin a & 0 \\ 0 & -K3 \sin b & (K3-K4)\sin b & K4 \sin b \\ 0 & 0 & -K4 \sin b & (K4-K5)\sin b \end{bmatrix} \qquad (5.13)$$

Treat T_{s1} and T_{f2} as fixed forces at segments I-II and II-III, respectively. Using Eq. (4.3-5) and replacing μ_s and μ_d with μ_{si} and l_{di}, calculate the unbalanced force, UBF_{II}. And then solve rl_1 by using Eq. (4.30) and replacing [Kf] with $[Kf]_i$. Notice that there is only one degree-of-freedom in this case. From Eq. (4.31), the tendon force T_{s1} and T_{s2} are updated by

$$T_{s1} = T_{s1} + K1\ rl_1$$

$$T_{s2} = T_{f2} - K2\ rl_1 \qquad (5.14)$$

5.3.1.2 Determine the sliding condition at node III. Compare T_{s2} and T_{f3}. If node III does not slide, no force adjustment is necessary. Otherwise, this is a two degrees-of-freedom distribution. Only node III has the unbalanced force, UBF_{III}. The unbalanced force at node II, UBF_{II}, is zero. After forming the unbalanced force matrix, solve rl_1 and rl_2 by using Eq. (4.30). The updated tendon forces are

$$T_{s1} = T_{s1} + K1\ rl_1$$

$$T_{s2} = T_{s2} + K2\ (rl_2 - rl_1) \qquad (5.15)$$

$$T_{s3} = T_{f3} - K3\ rl_2$$

5.3.1.3 Determine the sliding condition at node JJJ. Compare T_{s3} and T_{f4}. Again, if no sliding happens at node JJJ, then no refinement is made. Otherwise, calculate the unbalanced force at node JJJ. The unbalanced forces at node II and III are zero. After forming the unbalanced force matrix and slip stifness matrix, solve rl_1, rl_2 and rl_3 by using Eq. (4.30). The updated tendon forces are

$$\begin{aligned} T_{s1} &= T_{s1} + K1\ rl_1 \\ T_{s2} &= T_{s2} + K2\ (rl_2 - rl_1) \\ T_{s3} &= T_{s3} + K3\ (rl_3 - rl_2) \\ T_{s4} &= T_{f4} - K4\ rl_3 \end{aligned} \tag{5.16}$$

5.3.1.4 Determine the sliding condition at node JJ. Compare the updated force T_{s4} and T_{f5}. If no sliding happens, the effect of the slip loss reaches to node JJJ only and no refinement is necessary. If node JJ slides, then this is a four degrees-of-freedom case. Again, only node JJ has the unbalanced force. The slip displacement matrix, {rl}, can be solved by using Eq. (4.30). The initial forces after slip loss are

$$\begin{aligned} T_{s1} &= T_{s1} + K1\ rl_1 \\ T_{s2} &= T_{s2} + K2\ (rl_2 - rl_1) \\ T_{s3} &= T_{s3} + K3\ (rl_3 - rl_2) \\ T_{s4} &= T_{s4} + K4\ (rl_4 - rl_3) \\ T_{s5} &= T_{f5} - K5\ rl_4 \end{aligned} \tag{5.17}$$

5.3.2 Tensioning at Both Ends

Suppose the tendon is tensioned at both ends, then the tendon force T_{s5} and T_{s4} will equal to T_{s1} and T_{s2}, respectively. The same criterion is used here. But at most two nodes need to be checked (shown in Fig. 5.5).

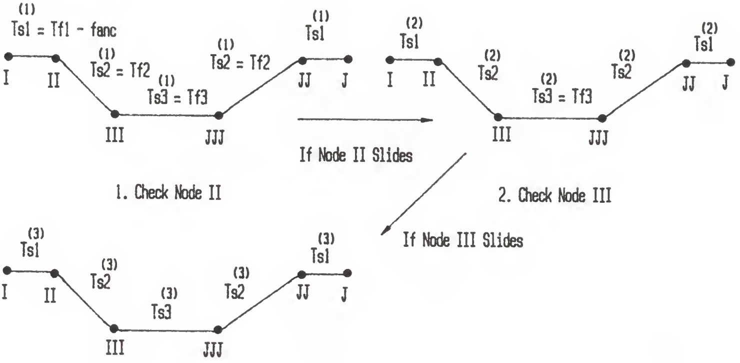


Fig. 5.5 Anchorage Slip Loss (Tensioning at Both Ends)

5.4 Computer Implementation

The tensioning process is added in subroutine INEL. After the instaneous losses are calculated, the initial force will be stored and retrieved in subroutine INIT. For the particular applications, including the tendon stiffness in the analysis will be the user's option. The procedures are done in subroutines STAT and STIF. The changes to the previous element routines are stated as follows:

5.4.1 Subroutine INEL11

The labeled common block /INFEL/ has been expanded:

```
 COMMON/INFEL/IMEM,KST,LM(18),NODE(6),INS(6),INSO(6),XX(6),
*             YY(6),ZZ(6),PROP(5),AREA,XL(5),B(7),SEL(5),
*             STOT(5),TT(3,3),T,TI(5),QL(12),SHA1,SHA2,
*             BK(4,4),SK(5),IP,IDUM,ANSL,SKP(324)
```

where PROP(5), TI(5), SHA1, SHA2, BK(4,4), SK(5), IP, IDUM and ANSL are added. The definitions are stated as follows:

PROP(1) = Young's modulus of the tendon.

PROP(2) = Static friction coefficient between the tendon and the concrete during tensioning.

PROP(3) = Dynamic friction coefficient between the tendon and the concrete during tensioning.

PROP(4) = Static friction coefficient between the tendon and the concrete.

PROP(5) = Dynamic friction coefficient between the tendon and the concrete.

TI(5) = The initial force of each segment after friction loss and anchorage slip loss.

SHA1 = $\mathrm{Sin}(90-\theta_1/2)$

SHA2 = $\mathrm{Sin}(90-\theta_2/2)$

BK(4,4) = The truss stiffness matrix. One part of the slip stiffness matrix.

SK(5) = The truss stiffness of each segment.

SHA1, SHA2, BK(4,4) and SK(5) were calculated in subroutine STAT and are shifted here to avoid repeat calculations.

IP = The loading segment number to include the tendon stiffness in the analysis.

This means the analysis will start including the tendon stiffness from the ip th loading segment. This number can be greater than the total loading segments and no tendon stiffness will be included in the analysis.

IDUM = Dummy interger variable.

ANSL = The amount of the anchorage slip.

The variables for the tensioning process are input after the material properties. These include NT, IP and ANSL, where NT is the index for the tensioning at one end or both two ends. The losses will be calculated after array INS(6), INSO(6) and QL(12) are initialized. The friction loss is calculated first, the procedures are

(1) Tell the number of ends at which the tendon is stretched during tensioning. If the tendon is tensioning at both ends, check the tendon profile's symmetry.

(2) Distinguish the tendon's profile, 4-node or 5-node or 6-node.

(3) Apply Eq. (5.6 -10) to calculate the tendon forces and store them in TI(5). If the tendon is not 6-node, the force adjustments are made.

After the friction loss has been calculated, the anchorage slip loss is then evaluated. The step-by-step method has been explained in Chapter 4 and section 5.3. The procedures used in the programming are

(1) Calculate the stiffness of each truss segment, SK(5), and initialize the loss, TLOSS, due to the slip.

(2) Form the slip stiffness matrix, SF.

(3) The loss, TLOSS, changes the tendon force of the segment I-II, TI(1), and induce the force redistribution. First, check the sliding condition of node II. If it slides, then check node III. The check will continue until the non-sliding node or the last node JJ is reached.

(4) Each sliding condition will induce the redistribution of the tendon force. The tendon forces will be determined based upon Eq. (5.14-17).

5.4.2 Subroutine INIT11

The common block /INFEL/ is changed to the one discussed in the above section. The variable, NSEG, indicating the number of the loading segment, is added to the common block /STLDPT/.

The other change in this subroutine is to replace the the force, T, with the initial force after losses, TI, in calculating the current force matrix, P. Next, the equivalent loading is determined by calling subroutine FORVER. The initial force matrix is shown in Eq. (4.25). The magnitude and the direction of the loading is shown in Fig. 5.6.

5.4.3 Subroutine STIF11

The tendon stiffness is excluded when the index, IP, is smaller than the current loading segment, NSEG. Even if the sliding code, INS(5), has been changed, and the subroutine is called to reform the stiffness, the tendon stiffness will stay zero under this situation. These statements are done at the beginning of the subroutine.

5.4.4 Subroutine STAT11

Except that the common blocks /INFEL/ and /STLDPT/ are changed, the common block /PRITC/ is added into this subroutine. The only variable used in this common block is the iteration number, NIT.

In the force solving process, the force, T, is replaced by the corresponding initial force, TI_i, in the ith segment.

When the index, IP, equals to the current loading segment, the tendon stiffness should be added to the total stiffness. At the beginning of the loading segment, i.e., the first iteration, the state code, KST, is automatically set to one. These statements are put in the end of the subroutine.

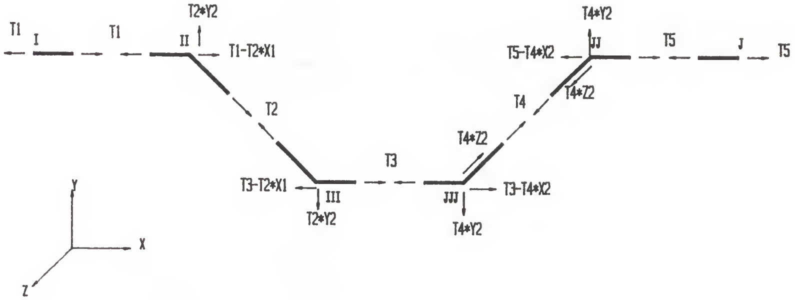


Fig. 5.6 Initial Force After Immediate Losses

CHAPTER 6
EXAMPLES AND PARAMETRIC STUDY

6.1 Introduction

Four examples are presented in this chapter to demonstrate the validity and applicability of the tendon element developed here. In order to find out the influence of the friction on the structural behavior, a parametric study regarding the cofficient of friction, the tendon force and external force is presented.

The first example is used to verify the tendon element without friction. The vertical displacement and the change of the tendon force are compared to those derived from beam theory. This example can be applied for the pretensioned prestressed concrete structure because no slip between the tendon and the concrete is allowed. The second example shows the friction loss and the seating loss during the tensioning process and the friction at the deviation block during the application of external loading. This example shows the use of external prestressing for the post-tensioned prestressed concrete structure. The third example shows the application for the continuous beam with external prestressing. The results are then compared to those from conventional analysis. The final example introduces the link element [22,23] to model the interface of the concrete segments in the segmental bridge with external prestressing.

The parametric study presents the effects of the different parameters to the whole structural behavior. The vertical displacements, the tendon forces and the gap widths of the segmental bridge due to the variations of the parameters are presented.

6.2 Example 1--For the Pretensioned Bonded Structure

A simple-supported beam with the harped tendon is adopted in this example. The configuration of the tendon and the dimensions of the beam are shown in Fig. 6.1. The material properties used for the analysis are shown in Table 6.1. The prestressed concrete beam is modeled by forty-eight shell elements [22,23] and one tendon element in the xy plane. To verify the finite element mesh and to account for the contribution of each loading, three loading segments are performed. The first loading segment is due to the prestressing force, the second one is due to the self-weight of the beam and the third one is due to one pair of concentrated loads at one-third of the span from each support.

Two kinds of the boundary conditions are used in this example, the first is used to match the assumption of beam theory and the end supports are put along the neutral axis of the cross section, the other model's end supports are

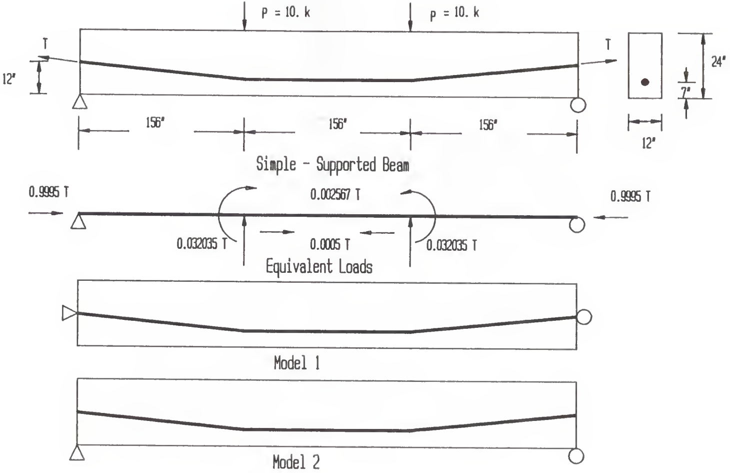


Fig. 6.1 Pretensioned Prestressed Concrete Beam--Example 1

put at the bottom of the cross section. In the first model, two cases are performed: one with the tendon stiffness and the other without the tendon stiffness, and the poisson's ratio is set to zero. In the second model, the tendon stiffness is considered and the poisson's ratio is set to 0.2.

6.2.1 Verification of Displacements

The theoretical value of the displacement at midspan due to each loading case is calculated from beam theory and can be stated as follows:

6.2.1.1 Camber due to the prestressing force. For the harped tendon used here, the camber is [15,16]

$$\Delta_i = 23/216(TeL^2/EI) \qquad (6.1)$$

in which

T = prestressing force.

e = eccentricity of the tendon to the neutral axis.

L = span length.

E = Young's modulus of concrete.

I = moment inertial of the net cross section
(w/o tendon stiffness).

= moment inertial of the transformed section
(w tendon stiffness).

6.2.1.2 Displacement due to the self-weight of concrete. The displacement due to the distributed self-weight is

$$\Delta_w = 5/384(wL^4/EI) \qquad (6.2)$$

in which w is the concrete weight per unit length.

6.2.1.3 Displacement due to the concentrated loads. The displacement due to the point loads (shown in Fig. 6.1) is expressed by

$$\Delta_p = 23/648(PL^3/EI) \tag{6.3}$$

6.2.2 Verification of the Change of the Tendon Force

(With Tendon Stiffness)

If the prestressed concrete beam is regarded as the composite structure of concrete and steel tendon, then the increment of the tendon force can be derived from the virtual work [24]

$$\Delta_{1e} + T_{inc}\,\Delta_{11} = 0 \tag{6.4}$$

in which

Δ_{1e} = the displacement along the tendon due to the external loads.

T_{inc} = the increment of the tendon force.

Δ_{11} = the displacement along the tendon due to the unit tendon force.

The equivalent loading of the tendon can be divided into two parts: one is flextural loads and the other is axial loads (shown in Fig. 6.2). Then

$$\begin{aligned}\Delta_{11} = {} & 2/EI[(5/162)\sin^2\alpha L^3 \\ & +(1-\cos\alpha)^2 e^2 L/6 \\ & +\sin\alpha(1-\cos\alpha)eL^2/9] \\ & +L/EA(2/3*\cos\alpha+1/3)+L/E_sA_s(1/3+\cos^2\alpha*2/3)\end{aligned} \tag{6.5}$$

where α is the sharp angle at the harped point.

For the self-weight of the beam, the displacement Δ_{1w} is

$$\Delta_{1w} = wL^3/EI[-11/486\ \sin\alpha\ L\ - 13/324(1-\cos\alpha)e] \tag{6.6}$$

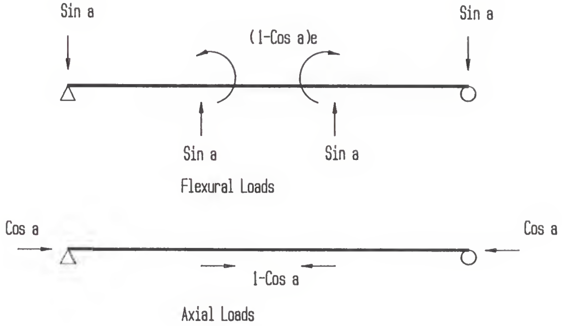


Fig. 6.2 Equivalent Loading of Unit Tendon Force--Example 1

For the point loads, the displacement α_{1p} is

$$\Delta_{1p} = (PL^2/EI)*[-5/81 \sin\alpha \; L - (1-\cos\alpha)e/9] \qquad (6.7)$$

Substitute Eq. (6.6-8) into Eq. (6.5), the change of the tendon force at each stage can be evaluated.

The results from the output and beam theory for each case are listed in Table 6.2. Compare model 1 without tendon stiffness to the ideal beam, the differences of the results are within 0.5%. For model 1 with tendon stiffness, the increment of the tendon force and the displacement, excluding the camber, for each loading are very close to those from the theory. The tendon stiffness has more influence in the camber due to the prestress, because the high axial loads are applied at the ends where the tendon stiffness is comparatively high. To demonstrate this, the equivalent loadings without the axial loads, matched to the ideal beam, are applied to the same beam and results in a higher camber. The tendon stiffness reduces the vertical displacement, except for the camber, about 1%. These results show the validity of this element. Model 2 considers the effects of the tendon stiffness and the poisson's ratio and the results are satisfactory compared to the ideal beam.

6.3 Example 2--For External Prestressing

For simplicity, the simple-supported beam with a rectangular cross section, shown in Fig. 6.3, is used in this example. The structure is a post-tensioned prestressed concrete beam and the harped tendon is unbonded with the

Table 6.1 Material Properties of Example 1

f'_c(ksi)	E_c(ksi)	w_c(k/ft^3)	v	E_s(ksi)	A_s(in^2)	F_i(k)
5.	4833	150	0 (0.2)	30000	2.5	300

* v=0.2 for model 2

Table 6.2 Results of Example 1

	Load* Case	Model 1 (w/o stiff)	Ideal Beam	Model 1 (w stiff)	Composite Beam	Model 2
Y Dis.	1	0.5263	0.5236	0.4818	0.5093	0.5016
	2	-0.2350	-0.23373	-0.2317	-0.22735	-0.2335
	1+2	0.2913	0.28987	0.2501	0.28195	0.2681
	3	-0.5475	-0.54455	-0.5392	-0.52968	-0.5434
	1+2+3	-0.2562	-0.25468	-0.2891	-0.24773	-0.2753
	Initial force = 300.kips					
Tendon Force	1	272.8	-	274.6	-	274.5
	δ1	27.2	-	25.4	-	25.5
	1+2	274.4	-	276.5	-	276.6
	δ2	1.6	-	1.9	1.96	2.1
	1+2+3	279.3	-	281.0	-	281.5
	δ3	4.9	-	4.5	4.57	4.9

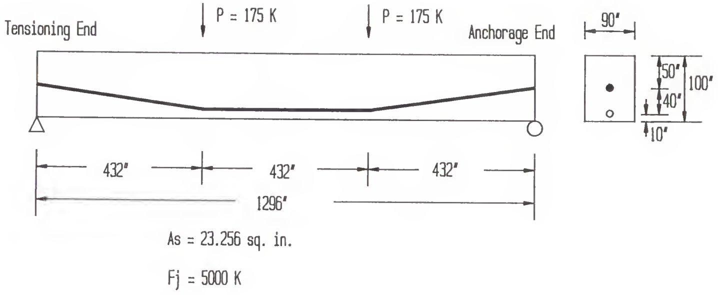


Fig. 6.3 External Prestressing Beam--Example 2

concrete except at the deviation blocks. The material properties are shown in Table 6.3. Different from the pretensioned beam, there are friction and anchorage slip losses during the tensioning. Friction at the deviation blocks will happen after the structure is deformed. The tendon stiffness is included in the analysis, because the tendon contacts with concrete at the deviation blocks.

Three cases are performed in this example. The first case is to show the validity of the instaneous prestress losses due to friction and anchorage slip. For post-tensioned prestressed concrete structures, jacking force at the tensioning end is input instead of the initial force. After the instaneous losses happened, the net tendon force is then applied to the structure. The friction, due to the deformation of the deviation block, is ignored by setting coefficient of friction to zero. Therefore, the tendon is continuous and the final tendon force is uniform along the length. The output is shown in Table 6.4. The verification of the friction loss can be performed by using Eq. (2.2). The tendon forces after the immediate losses are shown in Fig. 6.4.

The second case shows the application of the friction induced by the deformation of the structure. The initial losses are neglected, i.e., the initial force is assumed constant along the length. Two loading segments are used, the first one is the prestress and the self-weight of

Table 6.3 Material Properties of Example 2

	Case 1	Case 2 ***	Case 3 ***
f_c(ksi)	6.	-	-
E_c(ksi)	4415	-	-
w_c(lb/ft^3)	150	-	-
v	0.2	-	-
A_s(in^2)	23.256	-	-
E_s(ksi)	29000	-	-
F_i(kip)	4200.	N/A	N/A
F_j(kip)	5000.	4200.	5000.
μ_i*	0.2	0.0	0.2
μ_f**	0.0	0.3	0.3
δ_{anc}(in)	0.3	N/A	0.3

- the same as case 1

* μ_i - friction coefficient during tensioning.

** μ_f - friction coefficient after force transfer.

*** the tendon is tensioned at one end only.

Table 6.4 Output for Case 1 & Case 3 of Example 2

Jacking force = 5000. k μ_i =0.2 δ_{anc} =0.3 in.

		Initial Tendon Force		
		II-III	III-JJJ	JJJ-JJ
After friction loss		5000.	4909.	4819.
After seating loss		4677.	4764.	4819.
	Dis.	Final Tendon Force		
Load 1+2	.1256	4504.	4627.	4638.
Load 1+2+3	-.2853	4542.	4670.	4660.

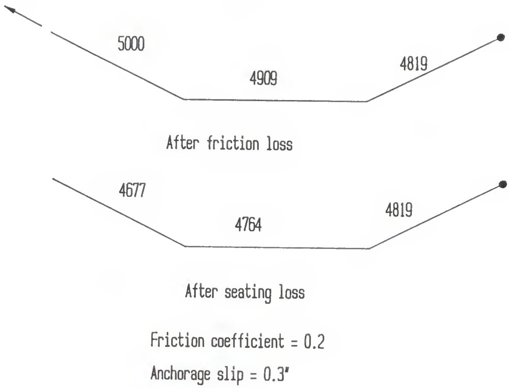


Fig. 6.4 Immediate Prestress Losses--Example 2

the beam and the second one is the external loads, shown in Fig. 6.3. The output, shown in Table 6.5, is compared to that with no friction.

The third case shows the combined effects of instaneous prestress losses and the friction loss due to the deformation of the structure. The results are shown in Table 6.4.

6.4 Example 3--For the Continuous Girder

This example shows the application of the five-node tendon, mostly used as external prestressing in continuous girders. The structure has two equal spans and is post-tensioned by external prestressing. The dimensions of the structure are shown in Fig. 6.5, and the material properties are shown in Table 6.6. Three tendons are used in each span and overlapped in the middle pier segment to assure the continuity of the structure. The configurations of the tendons in each span are arranged to be symmetrical to the middle support. Each tendon is tensioned at one end only and has the same amount of anchorage slip. The direction of the tensioning is also shown in Fig. 6.5.

Two loading segments are used here. The first is the prestress and the self-weight of the girder, the second is the external point loads, shown in Fig. 6.5. The results are listed in Table 6.7. For the purpose of comparison, the same structure is analyzed again by the conventional method in which the program, STAN [25], is used. In the

Table 6.5 Output--Case 2 of Example 2

	$\mu_f=0.3$				$\mu_f=0$	
		Tendon Force				
Load	Dis.	II-III	III-JJJ	JJJ-JJ	Dis.	Tendon force
1	.8820	4003.	3972.	4003.	.8840	3992.
1+2	.0083	4049.	4094.	4049.	.005625	4064.
1+2+3	-.4092	4070.	4152.	4070.	-.4065	4098.

*load 1--Prestress
load 2--Beam Weight
load 3--External loads

Table 6.6 Materials Properties of Example 3

f_c'	w_c	E_c	A_s	E_s	F_j	μ_i	μ_f	δ_{anc}
6.	150.	4415.	5.814	29000	1250	0.2	0.5	0.3

Table 6.7 Results of Example 3

Load Case	Displacement (in.)	
	ANSR	STAN
1. Prestress	.5903	.60554
2. Self-weight	-.49913	-.47749
3. External Loads	-.1527	-.14845
4. Total	-.06153	-.02040

*Displacement was measured at the midspan of the first span.

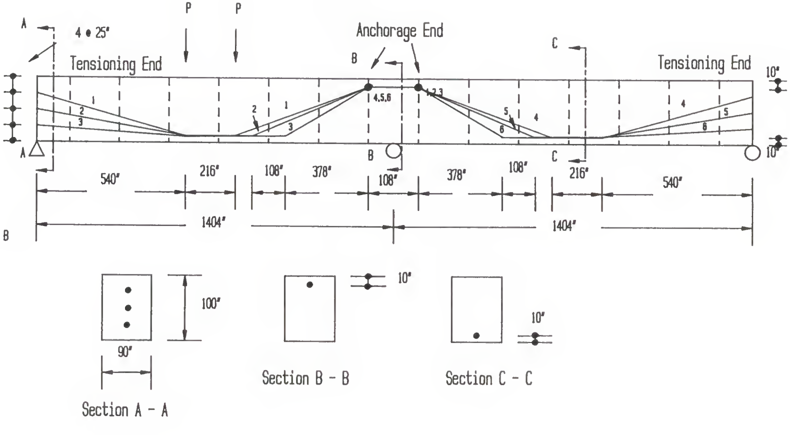


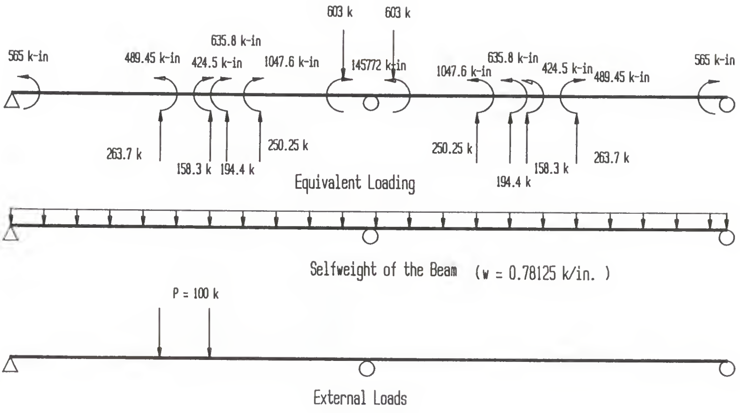
Fig 6.5 Continuous Beam--Example 3

conventional model, shown in Fig. 6.6, the beam elements are used and the tendon elements are replaced by the equivalent loadings. The friction loss and the anchorage slip loss during the tensioning are calculated due to the assumption that the amount of the anchorage slip is distributed along the tendon. No friction effects, due to the deformation of the structure, and no tendon stiffnesses are considered. The vertical displacements from the conventional analysis are included in Table 6.7.

6.5 Example 4--For the Segmental Bridge

This example has the same dimensions with the example used in section 6.3, but more tendons, with different configurations, are used to control the gaps between the segments. Link elements [22,23] are added along the interface between the segments in order to simulate the behavior of gap openings. The dimensions and the tendon profiles are shown in Fig. 6.7. The material properties are listed in Table 6.8.

Since the tendons are tensioned at both ends, the initial forces are symmetrical. Friction at the interface of the segments, modeled by link elements, is included. If the link elements are open, the structural stiffness will be reduced. The reduction of the stiffness depends upon the number of open link elements. To study the effects of the friction at the deviation blocks, two cases are investigated in this example. The first includes the friction while the second does not.



Equivalent Loading

Selfweight of the Beam (w = 0.78125 k/in.)

External Loads

Fig. 6.6 Structural Model Used for Comparison--Example 3

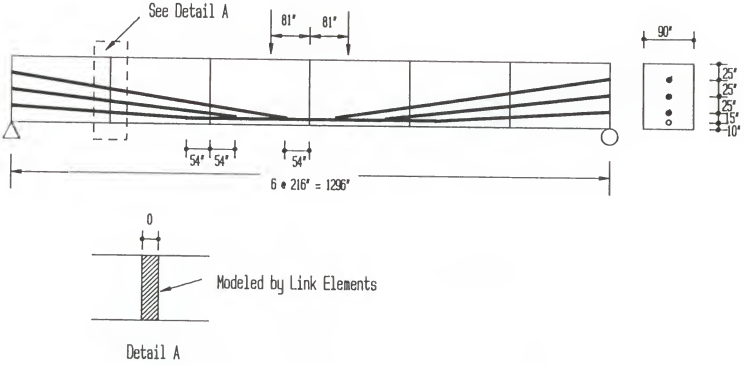


Fig. 6.7 Example for Segmental Bridge

The output, including the vertical displacement, the tendon force and the gap width, is shown in Table 6.9. The results indicate that the friction reduces not only the maxmium gap width but also the vertical displacement at the midspan. Therefore, the friction can reduce the possibility of gap openings and then increase the ultimate strength of the bridge.

6.6 Effects of the Friction

Due to the lack of any test data about the friction coefficient at the deviation block of external prestressing, the study uses the different values of the friction coefficient, ranged from 0. to 0.5, to find out the effects of friction on structural behavior. The structures of example 2 and 4, designated regular beam and segmental beam respectively, are used in this study.

For the structure of example 2, the following parameters are used:

1. Initial force 4200 kips (immediate losses are ignored).
2. External load from 0 to 500 kips.
3. Friction coefficient from 0. to 0.3.

Two loading segments are performed. Fig. 6.8 shows the load-deflection curves due to the various friction coefficients. Fig. 6.9 shows the plots of the tendon forces at midspan as functions of external loads due to the

Table 6.8 Material Properties of Example 4

	Case 1	Case 2
f_c(ksi)	6.	-
E_c(ksi)	4415	-
w_c(lb/ft^3)	150	-
v	0.2	-
A_s(in^2)	7.752	-
E_s(ksi)	29000	-
F_j(kip)	1650.	-
μ_i*	0.25	-
μ_f**	0.6	0.
μ_s***	0.3	-
δ_{anc}(in)	0.3	-

- the same as case 1
* μ_i - friction coefficient during tensioning.
** μ_f - friction coefficient after force transfer.
***μ_s - friction coefficient for link elements (shear transfer).
**** the tendon is tensioned at one end only.

Table 6.9 Results of Example 4

μ_f	Displacement	Max. Gap width	Tendon Force		
			1	2	3
0.	-0.5284	.001013	1534	1550	1568
0.6	-0.5232	.0005	1581	1571	1579
			(1519	1522	1546)*

*the tendon force at the tensioning end.

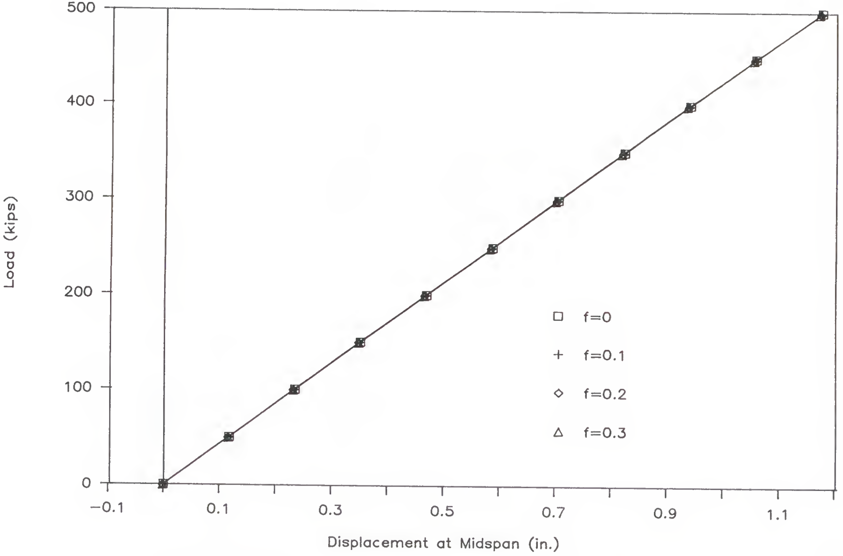


Fig. 6.8 Load - Deflection Curves for Various Friction Coefficients (Example 2)

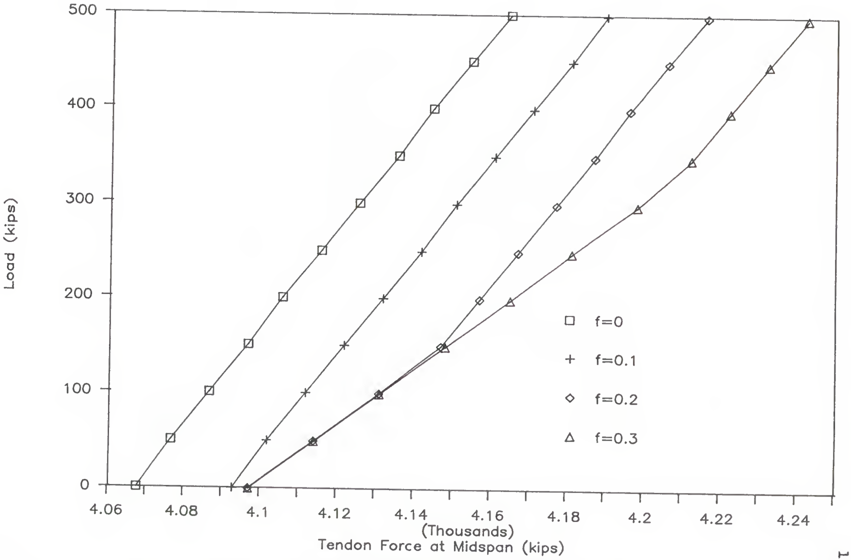


Fig. 6.9 Tendon Force at Midspan v.s. External Load for Various Friction Coefficients (Example 2)

different friction coefficients. The plots of the tendon forces at the tensioning end versus the external loads due to the various friction coefficients are shown in Fig. 6.10.

For the structure of example 4, the following parameters are used:

1. Jacking force 1250 kips for each tendon.
2. The tendons are tensioned at two ends.
3. External load from 180 to 228 kips.
4. Friction coefficient from 0 to 0.5.

Fig. 6.11 shows the load-deflection curves for the various friction coefficients. The maximum gap width verse the external load for the various friction coefficients is shown in Fig. 6.12.

It can be seen that the displacement decreases slightly with the increase of the friction coefficient. This can be explained by the difference between the tendon stiffness and the truss stiffness. Furthermore, the open of gaps can reduce the total stiffnesses of the structure, so the difference is more obvious in example 4 than in example 2. When the structure is under the specific exrternal load and the friction coefficient is big enough, then all the nodes of the tendon will not slide and the tendon behaves as the truss elements. At this time, the behavior of the tendon remains the same despite the increase of friction. Further, the increase of the external loads will reduce the

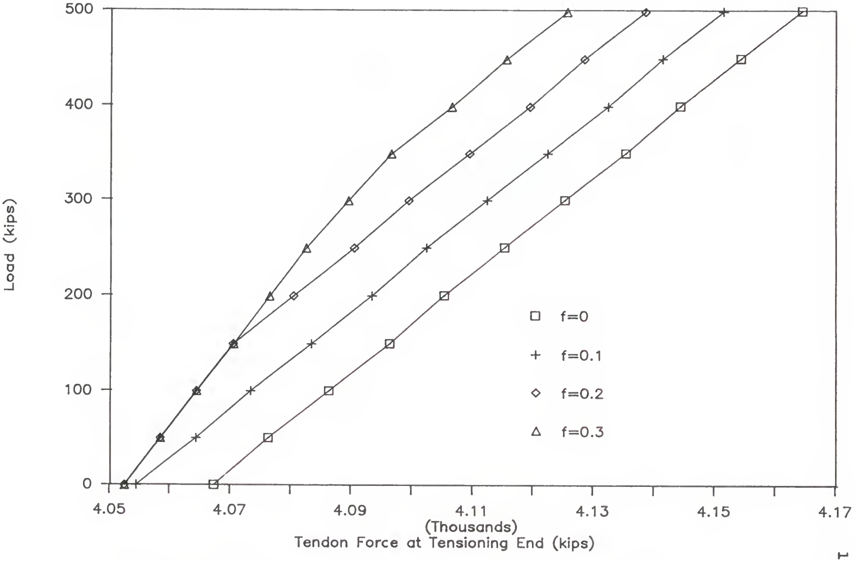


Fig. 6.10 Tendon Force at Tensioning End v.s. External Load for Various Friction Coefficients

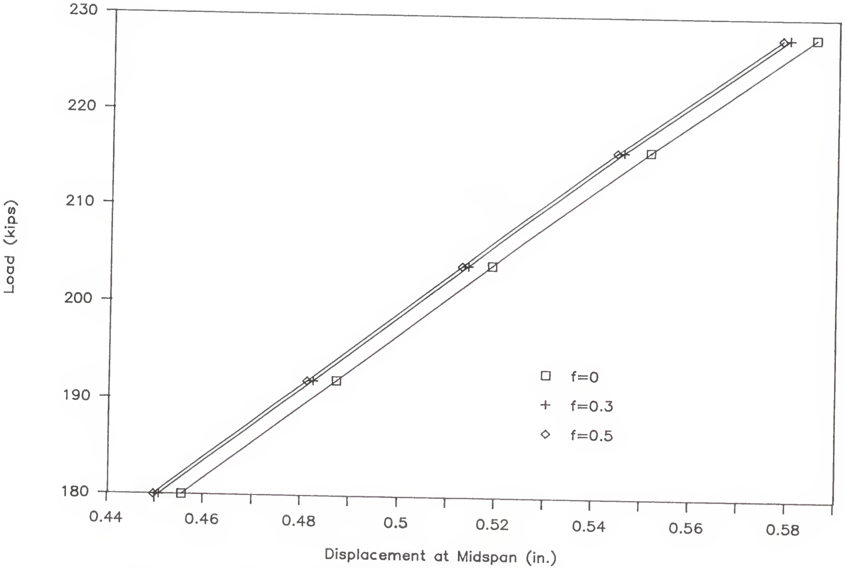


Fig. 6.11 Load - Deflection Curves for Various Friction Coefficients (Example 4)

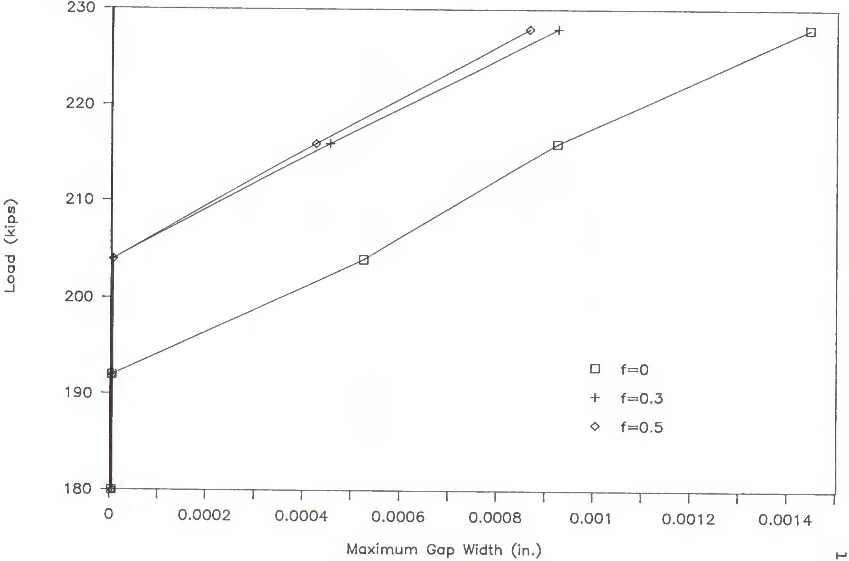


Fig. 6.12 Maximum Gap Width v.s. External Load for Various Friction Coefficients

resistant forces at the deviation blocks and cause the nodes to slide, and the tendon behaves as a continuous element. So the slope of the load-deflection curve will be slightly decreased due to the increase of the external loads. This tendency is more significant in example 4 than in example 2 due to the gap openings. Under the same external loads, the tendon force at midspan increases with the friction coefficient but at the same time the tendon force at the tensioning end decreases with the friction coefficient. This indicates that the friction increases the strength at the midspan but reduces the strength at the harped point. The sudden drop of the tendon force should be noticed especially the friction is big enough. From Fig. 6.12, it can be cleary seen that the friction helps reduce the maximum gap width by about 100% of that with no friction.

The results point to the importance of the friction at the deviation blocks for the prestressed non-segmental and segmental bridges.

CHAPTER 7
CONCLUSIONS AND RECOMMENDATIONS

7.1 Conclusions

7.1.1 Tendon Element Without Friction

The tendon element without friction has been verified and proved to be useful in modeling the prestressing reinforcement for both the pretensioned and the post-tensioned prestressed concrete structures.

The formation of the tendon stiffness is based upon the continuity along the whole tendon which is the major difference with the formation of truss element or frame element. Consequently, this is a more reasonable approach than the conventional method. Further, considering the tendon stiffness is more plausible in the analysis of the bonded prestressed concrete structures.

The initial force of the tendon is automatically transfered to the structure in the form of the equivalent loading. This is another advantage of the element, it avoids having to calculate the equivalent loading, especially when many tendons are used. For the tendon element without friction, the initial force is constant. This matches the behavior of the pretensioned concrete beams and is still applicable to the bonded post-tensioned structures with the proper estimate for the immediate losses.

The prestress loss due to the deformation of the structure is evaluated from the change in the geometry of the tendon. Since the angle change is very small, the calculation is according to the change of the total tendon length. The final tendon force is then calculated by deducting the loss from the initial force.

The applicability of the element has been verified in Chapter 6. The displacement, due to the prestress, and the increment of the tendon force, due to external loading, proved to be satisfactory for the analysis of prestressed concrete structures.

7.1.2 Tendon Element With Friction

The element was developed to simulate the external prestressing reinforment used in post-tensioned prestressed concrete structures. Immediate prestress losses, such as friction loss and anchorage loss, are included.

The friction loss is much less in external prestresssing than that in internal prestressing because of the less contact area between steel and concrete. The simplied formula presented here proved to be adequate compared to the more frequently used formula. The anchorage loss can be predicted by using the step-by-step method proposed in Chapter 5. The reverse friction, caused by the seating loss, is solved by the slip stiffness method proposed in this study.

The formation of the stiffness matrix is strongly dependent upon the structural model of the prestressing steel. Because of the existence of friction, the tendon element may not be one continuous element but a combination of truss elements and tendon elements. The structural model is determined by the sliding condition at each node. Since the element is nolinear, the stiffness should be updated if the structural model is changed.

The initial force is not constant along the tendon due to the instaneous prestress losses. After the immediate losses have been calculated, the equivalent loading is then formed and transfered to the concrete.

The slip stiffness was developed to solve the tendon force with regard to the friction at the deviation block. The formation is basically the combination of the truss stiffness for the segmental members of the tendon and the friction stiffness at the deviation block. The degrees-of-freedom of the element are determined by the structural model. The unbalanced load is determined by the equilibrium at each degree-of-freedom. The final tendon force is then solved by the force distribution inside the tendon element.

Although the friction at the deviation block has not been investigated, the method used here is an available approach to verify the importance of the friction. The results presented in Chapter 6 have shown the effects of the friction at the deviation blocks, especially for the segmental bridges. It can be concluded that enough friction at the deviation block increases the tendon stiffness and help reduce the maximum gap width and increases the strength of the segmental bridge.

7.2 Recommendations

The following recommendations are made based upon the results of this study:

1. The tendon element without friction can provide good results for the analysis of the prestressed concrete structure with bonded tendons.
2. External tendon at the deviation block of the segmental bridges should be well bonded to provide enough force to resist gap opening.
3. The external tendon used in the non-segmental structure can be unbonded at the deviation block. But the concrete block should be designed to resist the high tension of the tendon.
4. The force jump of the external tendon at the deviation block due to the friction should be taken into account in design.
5. The profile of the external tendon is better arranged to form the bigger angle at the harped point to provide larger friction. But the high tension at the block should be considered.
6. The location of the deviation block should be well

distributed along the concrete segments in order to control the gap opening in segmental bridges.

7.3 Future Work

Convergence problems have been encountered when the tendon element (with friction) and link elements are used at the same time. A possible reason is due to the nonlinearities of both elements, the nonlinear analysis method used in ANSR is not able to find the right convergence path. The improvement of the convergence process will be valuable for the analysis of the segmental bridges.

The long-time prestress losses due to creep and shinkage of concrete and relaxation of the prestressing steel are not included here. If the shell element has the capability of handling creep and shinkage, then the prestress losses can be done without any change in this tendon element. If the convergence can be improved, the loss due to relaxation of the prestressing steel can also be added.

The structures used in the examples were modeled by shell elements in the x-y plane only. This is because there was difficulty in modeling the box section with the existing shell elements. The displacement from the model didn't match that from beam theory. Hopefully, the improvement of shell elements can help make box section model more reasonable.

Due to the lack of experimental data for the friction at the deviation block, the method proposed here can not be verified and compared. If experiments, measuring the friction coefficient and the variation of the tendon force, can be conducted, it will help understand the effects of friction.

APPENDIX A

USERS MANUAL FOR THE TENDON ELEMENT

TENDON ELEMENT USER'S GUIDE

A.1 CONTROL INFORMATION - One Card

Columns	Notes	Name	Data
1-5(I)		NGR	Element group indicator.
6-10(I)		NLES	Number of elements in group.
11-15(I)		MFST	Element number of first element in group.
16-25(F)		DKO	Initial stiffness damping factor, β_0.
26-35(F)		DKT	Current tangent stiffness damping factor, β_t.
41-80(A)		GRHED	Optional group heading.

A.2 MATERIAL PROPERTY INFORMATION

A.2(a) Control Card

Columns	Notes	Name	Data
1-5(I)		NMAT	Number of different material types. Default=1

A.2(b) Properties - NMAT set of cards

Columns	Notes	Name	Data
1-5(I)		M	Material Number.
*6-15(F)		E	Young's modulus of elasticity.
*16-25(F)		μ_{is}	Static friction coefficient between duct and tendon.
*26-35(F)		μ_{id}	Dynamic friction coefficient between duct and tendon.
*36-45(F)		μ_{fs}	Static friction coefficient between concrete and tendon.
*46-55(F)		μ_{fd}	Dynamic friction coefficient between concrete and tendon.

*A.2(c) Tensioning method & Anchorage slip length

1-5(I)	nt	Index for tensioning at one end or both two ends. = 1 one end = 2 two ends
6-10(I)	ip	The load segment number from which the analysis starts including the tendon stiffness.
11-20(F)	ansl	Anchorage slip length due to the prestressing method.

A.3 ELMENT DATA - one card

Columns	Notes	Name	Data
1-5(I)		N	Element number
6-10(I)		NODC	Node number I
11-15(I)		NODC	Node number II
16-20(I)		NODC	Node number III
21-25(I)		NODC	Node number JJJ
26-30(I)		NODC	Node number JJ
31-35(I)		NODC	Node number J
36-40(I)		MT	Material type number
41-50(F)		AO	Area of tendon
51-60(F)		T	Initial tension of tendon
62-66(I)		ND	Node number differences

*For the version without friction, the specified items or section should be omitted.

APPENDIX B

LISTINGS OF THE TENDON ELEMENT WITHOUT FRICTION

```
C-----------------------------------------------------------------------
      SUBROUTINE INEL11 (NJT,NDKOD,X,Y,Z,KEXEC)
C-----------------------------------------------------------------------
C     *****************************************************
C     SUBROUTINE TO READ, GENERATE AND PRINT ELEMENT DATA.
C     ELEMENT TYPE = 11 THREE DIMENSIONAL TENDON ELEMENTS.
C     *****************************************************
C
      IMPLICIT DOUBLE PRECISION(A-H,O-Z)
      DIMENSION NDKOD(NJT,3),X(NJT),Y(NJT),Z(NJT),ICOM(756)
      COMMON /TAPES / NIU,NOU,NT1,NT2,NT3,NT4,NT5,NT6,NT7,NT8,
     *                NT9,NT10,NT11,NT12,NT13,NT14,NT15,NT16
      COMMON /INFGR / NGR,NELS,MFST,IGRHED(10),NINFC,LSTAT,
     *                LSTF,LSTC,NDOF,DKO,DKT,EPROP(1000)
      COMMON /INFEL / IMEM,KST,LM(18),NODE(6),XX(6),YY(6),
     *                ZZ(6),PROP(1),AREA,XL,B(7),SEL,VTOT,
     *                STOT,TT(3,3),T,SKP(324)
      COMMON /PRECIS/ ZERO,ONE
      COMMON /WORK  / NODC(6),XE(6),YE(6),ZE(6)
      EQUIVALENCE (IMEM,ICOM(1))
C
      DIMENSION IAST(3)
      DATA IAST /2H  ,2H *,2H**/
C
C     ---------------------------------------------------------
C     MEANINGS OF VARIABLES IN COMMON BLOCK /INFEL/
C
C     IMEM      - ELEMENT NUMBER.
C     KST       - STIFFNESS UPDATE CODE.
C     LM(18)     - LOCATION MATRIX.
C     NODE(6)   - NODE NUMBERS FROM END I TO J.
C     KGEOM     - LARGE DISPLACEMENT CODE.
C     KTHO      - RESPONSE OUTPUT CODE.
C     KBUCK     - BUCKLING BEHAVIOR CODE.
C     KOD       - CURRENT YIELD CODE.
C     PROP(1)   - MATERIAL PROPERTIES.
C     AREA      - CROSS SECTIONAL AREA.
C     XL        - ELEMENT LENGTH.
C     XYZ(3,6)  - COORDINATES OF NODE I TO J.
C     B(8)      - DIRECTION PARAMETERS.
C     TT(3,3)   - TRANSFORMATION MATRIX.
C     VTOT      - TOTAL AXIAL DEFORMATION.
C     STOT      - TOTAL STRESS
C     SEP       - AXIAL FORCE IN ELASTO-PLASTIC COMPONENT.
C     SEL       - AXIAL FORCE IN ELASTIC COMPONENT.
C     VENP, VENN
C               - POS. AND NEG. ENVELOPE VALUES OF AXIAL
C                 DEFORMATION.
C     VPACP, VPACN
C               - ACCUMULATED POS. AND NEG. PLASTIC DEFORMATIONS.
C     VBUCK     - BUCKLING DEFORMATION.
C     SENP, SENN
```

```
C               - POS. AND NEG. ENVELOPE VALUES OF AXIAL FORCE.
C     TVENP, TVENN
C               - TIMES AT POS. AND NEG. ENVELOPE VALUES OF
C                 DEFORMATION.
C     TSENP, TSENN
C               - TIMES AT POS. AND NEG. ENVELOPE VALUES OF AXIAL
C                 FORCE.
C     TOLY      - OVERSHOOT TOLERANCE FOR YIELD EVENTS
C     TOLU      - OVERSHOOT TOLERANCE FOR UNLOADING EVENTS
C     KODP      - PREVIOUS YIELD CODE.
C     LDUM      - INTEGER VARIABLE NOT USED
C     SKP(324)  - CURRENT TANGENT OR EFFECTIVE STIFFNESS MATRIX.
C     ------------------------------------------------------------
C
C     ELEMENT GROUP PARAMETERS
C
      IF (MFST.LE.0) MFST = 1
C
      NDOF  = 18
      LSTAT = 108
      LSTF  = 648
      LSTC  =   0
      NINFC = 756
      KST   = 1
C
      DO 100 I=3,756
100   ICOM(I) = 0
C
C     PRINT ELEMENT GROUP INFORMATION
C
      WRITE(NOU,2000) NGR,IGRHED,NELS,MFST,NDOF,NINFC,DKO,DKT
C
C     READ AND PRINT MATERIAL PROPERTIES
C
      READ (NIU,1000) NMAT
      IF (NMAT.LE.0) NMAT = 1
C
      WRITE(NOU,2010) NMAT
      DO 110 I=1,NMAT
110   READ (NIU,1005) M,EPROP(M)
      WRITE(NOU,2020) (M,EPROP(M),M=1,NMAT)
C
C     READ, GENERATE AND PRINT ELEMENT DATA
C
      IMEM  = MFST
      NLAST = MFST + NELS - 1
      WRITE(NOU,2030)
C
140   READ (NIU,1010) N,NODC,MT,AO,T,ND
C
      IF (MT.LE.0) MT = 1
      IF (ND.EQ.0) ND = 1
```

```
C
150    IF (N - IMEM) 155,160,210
C
155    JFAST  = IAST(3)
       KEROR = 1
       WRITE(NOU,2040) JFAST,N,NODC,MT,AO,T
       GO TO 290
C
160    DO 170 I=1,6
170    NODE(I) = NODC(I)
C
       KEROR = 0
       NDIF  = ND
       MTYP  = MT
       AREA  = AO
c       FORCE = T
       SEL=t
       JFAST = IAST(1)
C
       IF (N.NE.NLAST) GO TO 140
       GO TO 210
C
190    DO 200 I=1,6
200    NODE(I) = NODE(I) + NDIF
       JFAST = IAST(2)
C
210    WRITE(NOU,2050)JFAST,IMEM,NODE,MTYP,AREA,SEL
C
C      ELEMENT CONNECTIVITY AND DIRECTION
C      TRANSFORMATION OF COORDINATES
C
       DO 220 I=1,6
       NOD = NODE(I)
       II = 3*I
       CALL NCODLM(LM(II-2),NJT,NOD,1)
       CALL NCODLM(LM(II-1),NJT,NOD,2)
       CALL NCODLM(LM(II),NJT,NOD,3)
       XE(I)=X(NOD)
       YE(I)=Y(NOD)
220    ZE(I)=Z(NOD)
       CALL TRAN1(XE,YE,ZE,XX,YY,ZZ,TT)
C
       Z4=ZZ(5)-ZZ(4)
       DZ4=DABS(Z4)
       DX1=XX(2)-XX(1)
       IF(XX(1).EQ.XX(2)) DX1=ZERO
       DX2=XX(3)-XX(2)
       DX3=XX(4)-XX(3)
       DX4=XX(5)-XX(4)
       DX5=XX(6)-XX(5)
       IF(XX(6).EQ.XX(5)) DX5=ZERO
       DY2=YY(2)-YY(3)
```

```
      DY4=YY(5)-YY(4)
      XL1=DX1
      XL2=SQRT(DX2*DX2+DY2*DY2)
      XL3=DX3
      XLH45=SQRT(DX4*DX4+DZ4*DZ4)
      XL45=XLH45*XLH45+DY4*DY4
      XL4=SQRT(XL45)
      XL5=DX5
      XL=XL1+XL2+XL3+XL4+XL5
      BX=EPROP(MT)*AO/XL
      CY1=DY2/XL2
      SY1=DX2/XL2
      CY2=DY4/XL4
      CX2=DX4/XL4
      CZ2=Z4/XL4
      SX11=1. -SY1
      CX22=1. -CX2
C
C     INITIALIZE DIRECTION PARAMETERS
C
C
      B(1)=CY1
      B(2)=CY2
      B(3)=SY1
      B(4)=CX2
      B(5)=CZ2
      B(6)=SX11
      B(7)=CX22
      PROP(1) =BX
C
C     COMPUTE STIFFNESS MATRIX PROFILE AND TRANSFER
C              ELEMENT DATA TO TAPE
      CALL BAND(LM,NDOF,NDKOD)
      CALL COMPAC
C
C     CHECK LAST ELEMENT
C
      IF (IMEM.EQ.NLAST) GO TO 290
      IMEM = IMEM + 1
      IF (IMEM.EQ.N) GO TO 160
      GO TO 190
C
C     ERROR AND ELEMENT DATA GENERATION MESSAGES
C
290   IF (KEROR.EQ.0) GO TO 300
C
      WRITE(NOU,2060)
      CALL EXIT
C
300   RETURN
C
1000  FORMAT (I5)
```

```
 1005  FORMAT (I5,F10.0)
 1010  FORMAT (8I5,2F10.0,1X,I5)
 2000  FORMAT (26H ELEMENT GROUP INDICATOR = I3,
     *    36H (THREE DIMENSIONAL TENDON ELEMENTS)//5X,10A4//
     *    5X,43HNUMBER OF ELEMENTS IN THIS GROUP ........ = I5/
     *    5X,43HNUMBER OF FIRST ELEMENT IN THIS GROUP ... = I5//
     *    5X,43HNUMBER OF DEGREES OF FREEDOM PER ELEMENT  = I5/
     *    5X,43HLENGTH OF ELEMENT INFORMATION ARRAY ..... = I5//
     *    5X,43HDAMPING COEFFICIENT, BETA-O ............. = F11.5/
     *    5X,43HDAMPING COEFFICIENT, BETA-T ............. = F11.5)
 2010  FORMAT (//20H MATERIAL PROPERTIES//
     *         5X,36HNUMBER OF DIFFERENT MATERIAL TYPES = I5//
     *         5X,7HMAT.NO.,5X,7HMODELUS/)
 2020  FORMAT (I9,E12.5)
 2030  FORMAT (///20H ELEMENT INFORMATION//
     *    5X,4HELEM,5X,4HNODE,5X,4HNODE,5X,4HNODE,5X,4HNODE,
     *    5X,4HNODE,5X,4HNODE,5X,4HMAT.,8X,4HAREA,5X,7HINITIAL/
     *    5X,4H NO.,5X,4H   I,5X,4H  II,5X,4H III,5X,4H JJJ,
     *    5X,4H  JJ,5X,4H   J,5X,4H NO.,17X,7H FORCE /)
 2040  FORMAT (1X,A2,I6,7I9,2F12.4)
 2050  FORMAT (1X,A2,I6,7I9,2F12.4)
 2060  FORMAT (///15H ERROR MESSAGES//
     * 5X,3HKEY,5X,29H------------- MESSAGE -------,5X,5HERROR//
     * 5X,3H   ,5X,29HELEMENT DATA INPUT DIRECTLY  ,5X,5HNONE /
     * 5X,3H  *,5X,29HELEMENT DATA GENERATED       ,5X,5HNONE /
     * 5X,3H **,5X,29HELEMENT CARD OUT OF SEQUENCE ,5X,5HFATAL/)
       END
C**********************************************************
C      SUBROUTINE TRAN1-TRANSFORMATION of COORDINATES      *
C**********************************************************
       SUBROUTINE TRAN1(X,Y,Z,XX,YY,ZZ,T)
       IMPLICIT REAL*8 (A-H,O-Z)
       DIMENSION x(6),y(6),z(6),T(3,3),XX(6),YY(6)
      *          ,zz(6),V(4,4)
       CALL VECTOR(V(1,1),X(3),Y(3),Z(3),X(4),Y(4),
      * Z(4))
       CALL VECTOR(V(1,4),X(3),Y(3),Z(3),X(1),Y(1),Z(1))
       CALL CROS(V(1,1),V(1,4),V(1,3))
       CALL CROS(V(1,3),V(1,1),V(1,2))
          DO 100 K=1,3
          DO 100 L=1,3
  100 T(K,L)=V(L,K)
          DO 200 N=1,6
       XX(N)=T(1,1)*X(N)+T(1,2)*Y(N)+T(1,3)*Z(N)
       YY(N)=T(2,1)*X(N)+T(2,2)*Y(N)+T(2,3)*Z(N)
       ZZ(N)=T(3,1)*X(N)+T(3,2)*Y(N)+T(3,3)*Z(N)
  200     CONTINUE
       RETURN
       END
C------------------------------------------ VECTOR
       SUBROUTINE VECTOR (V,XI,YI,ZI,XJ,YJ,ZJ)
       IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION V(4)
      X=XJ-XI
      Y=YJ-YI
      Z=ZJ-ZI
      V(4)=SQRT(X*X+Y*Y+Z*Z)
      V(3)=Z/V(4)
      V(2)=Y/V(4)
      V(1)=X/V(4)
      RETURN
      END
C------------------------------------------------- CROSS
      SUBROUTINE CROS(A,B,C)
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION A(4),B(4),C(4)
      COMMON /IOLIST/ NTM,NTR,NIN,NOT,NT1,NFL,NT2,NT3,NT4,NT5
C-------------------------------------------------------
      X=A(2)*B(3)-A(3)*B(2)
      Y=A(3)*B(1)-A(1)*B(3)
      Z=A(1)*B(2)-A(2)*B(1)
      XYZ = X*X + Y*Y + Z*Z
      IF(XYZ.GT.0.0) GO TO 100
      WRITE (NTM,2000)
      WRITE (NOT,2000)
      STOP
  100 C(4)=SQRT(XYZ)
      C(3)=Z/C(4)
      C(2)=Y/C(4)
      C(1)=X/C(4)
      RETURN
C
 2000 FORMAT (' ** K NODE IS ALONG ELEMENT SEE MANUAL **'/)
      END
C-------------------------------------------------------------
      SUBROUTINE STIF11(ISTEP,NDF,CDKO,CDKT,FK,INDFK,ISTFC)
C-------------------------------------------------------------
C     *******************************************************
C     SUBROUTINE TO COMPUTE ELEMENT TANGENT STIFFNESS MATRIX.
C     ELEMENT TYPE = 11 THREE DIMENSIONAL TENDON  ELEMENTS.
C     *******************************************************
C
      IMPLICIT DOUBLE PRECISION(A-H,O-Z)
      DIMENSION FK(NDF,NDF)
      COMMON /INFGR / NGR,NELS,MFST,IGRHED(10),NINFC,LSTAT,
     *                LSTF,LSTC,NDOF,DKO,DKT,EPROP(1000)
      COMMON /INFEL / IMEM,KST,LM(18),NODE(6),XX(6),YY(6),
     *                ZZ(6),PROP(1),AREA,XL,B(7),SEL,VTOT,
     *                STOT,TT(3,3),T,SKP(324)
      COMMON /WORK  / S(12,12),SK(18,18)
      COMMON /PRECIS/ ZERO,ONE
c---- set to full matrix version
      indfk=0
C
```

```
C     LINEAR PART OF EFFECTIVE STIFFNESS
C
C-----TENDON(I,J) OR TENDON(I,JJ) OR TENDON(II,JJ) OR
C     TENDON(II,J)
      DO 10 I=1,12
      DO 10 J=I,12
10    S(I,J)=ZERO
      S(1,1)=ONE
      S(1,2)=-B(6)
      S(1,3)=-B(1)
      S(1,4)=-S(1,2)
      S(1,5)=-S(1,3)
      S(1,6)=-B(7)
      S(1,7)=B(2)
      S(1,8)=B(5)
      S(1,9)=-S(1,6)
      S(1,10)=-S(1,7)
      S(1,11)=-S(1,8)
      S(1,12)=-S(1,1)
      S(2,2)=B(6)*B(6)
      S(2,3)=B(6)*B(1)
      S(2,4)=-S(2,2)
      S(2,5)=-S(2,3)
      S(2,6)=B(6)*B(7)
      S(2,7)=-B(6)*B(2)
      S(2,8)=-B(6)*B(5)
      S(2,9)=-S(2,6)
      S(2,10)=-S(2,7)
      S(2,11)=-S(2,8)
      S(2,12)=B(6)
      S(3,3)=B(1)*B(1)
      S(3,4)=-B(1)*B(6)
      S(3,5)=-S(3,3)
      S(3,6)=B(1)*B(7)
      S(3,7)=-B(1)*B(2)
      S(3,8)=-B(1)*B(5)
      S(3,9)=-S(3,6)
      S(3,10)=-S(3,7)
      S(3,11)=-S(3,8)
      S(3,12)=B(1)
        DO 100 I=4,12
100   S(4,I)=-S(2,I)
        DO 200 I=5,12
200   S(5,I)=-S(3,I)
      S(6,6)=B(7)*B(7)
      S(6,7)=-B(7)*B(2)
      S(6,8)=-B(7)*B(5)
      S(6,9)=-S(6,6)
      S(6,10)=-S(6,7)
      S(6,11)=-S(6,8)
      S(6,12)=B(7)
      S(7,7)=B(2)*B(2)
      S(7,8)=B(2)*B(5)
      S(7,9)=-S(6,7)
      S(7,10)=-S(7,7)
      S(7,11)=-S(7,8)
      S(7,12)=-B(2)
      S(8,8)=B(5)*B(5)
      S(8,9)=-S(6,8)
      S(8,10)=-S(7,8)
      S(8,11)=-S(8,8)
      S(8,12)=-B(5)
        DO 300 I=9,12
300   S(9,I)=-S(6,I)
        DO 400 I=10,12
400   S(10,I)=-S(7,I)
      S(11,11)=-S(8,11)
      S(11,12)=-S(8,12)
      S(12,12)=ONE
      IF(XX(1).EQ.XX(2)) THEN
C-----FOR TENDON(II-JJ,J)
        DO 500 I=1,12
500   S(1,I)=ZERO
      S(2,2)=B(3)*B(3)
      S(2,3)=-B(3)*B(1)
      S(2,4)=B(3)*B(6)
      S(2,5)=-S(2,3)
      S(2,6)=-B(3)*B(7)
      S(2,7)=B(3)*B(2)
      S(2,8)=B(3)*B(5)
      S(2,9)=-S(2,6)
      S(2,10)=-S(2,7)
      S(2,11)=-S(2,8)
      S(2,12)=-B(3)
      ENDIF
C-----FOR TENDON(I,II-JJ)
550   IF(XX(5).EQ.XX(6)) THEN
        DO 600 I=1,12
600   S(I,12)=ZERO
      S(1,9)=-B(4)
      S(2,9)=B(4)*B(6)
      S(3,9)=B(4)*B(1)
      S(4,9)=-S(2,9)
      S(5,9)=-S(3,9)
      S(6,9)=B(4)*B(7)
      S(7,9)=-B(4)*B(2)
      S(8,9)=-B(4)*B(5)
      S(9,9)=B(4)*B(4)
      S(9,10)=-S(7,9)
      S(9,11)=-S(8,9)
      ENDIF
C-----FOR TENDON(II-JJ)
650   IF(XX(1).EQ.XX(2).AND.XX(5).EQ.XX(6)) THEN
      S(1,9)=ZERO
      S(2,9)=-B(4)*B(3)
      ENDIF
        DO 700 K=1,12
        DO 700 L=1,12
        S(L,K)=S(K,L)
 700  CONTINUE
      DO 750 K=1,12
      DO 750 L=1,12
        S(L,K)=S(L,K)*PROP(1)
 750  CONTINUE
      CALL TRAN2(TT,S,SK,18,1)
      IJ =0
      DO 800 I=1,NDOF
      do 800 j=1,ndof
      IJ       = IJ + 1
      STIF     = SK(I,J)
      FK (I,J) = STIF
c      FK (J,I) = STIF
800   SKP(IJ)  = STIF
C
      RETURN
      END
C--------------------------------------------------------TRAN2
      SUBROUTINE TRAN2(T,SL,S,ND,NL)
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION T(3,3),TG(12,18),SL(12,12),S(18,18),ST(12,18)
      COMMON/PRECIS/ ZERO,ONE
      DO 50 I=1,12
      DO 50 J=1,18
      ST(I,J)=ZERO
50    TG(I,J)=ZERO
      DO 60 I=1,18
      DO 60 J=1,18
60    S(I,J)=ZERO
C-----FORM THE TRANSFORMATION MATRIX TG
      DO 100 J=1,3
100   TG(1,J)=T(1,J)
      DO 150 I=2,3
      DO 150 J=4,6
       II=I-1
       JJ=J-3
150   TG(I,J)=T(II,JJ)
      DO 200 I=4,5
      DO 200 J=7,9
       II=I-3
       JJ=J-6
200   TG(I,J)=T(II,JJ)
      DO 250 L=6,9,3
      DO 250 I=L,L+2
      DO 250 J=L+4,L+6
       II=I-L+1
       JJ=J-L-3
```

```
250   TG(I,J)=T(II,JJ)
      DO 300 J=16,18
       JJ=J-15
300   TG(12,J)=T(1,JJ)
C-----EVALUATION ST=SL*TG
      DO 400 I=1,12
      DO 400 J=1,18
      DO 400 K=1,12
400   ST(I,J)=ST(I,J)+SL(I,K)*TG(K,J)
C---- S=TTG*SL*TG
      DO 500 I=1,18
      DO 500 J=I,18
      DO 500 K=1,12
500   S(I,J)=S(I,J)+TG(K,I)*ST(K,J)
      DO 600 I=1,18
      DO 600 J=I,18
600   S(J,I)=S(I,J)
      RETURN
      END
C-----------------------------------------------------
      SUBROUTINE STAT11 (NDF,Q,TIME,FACAL,FACIL,ALFA)
C-----------------------------------------------------
C     ***************************************************
C     SUBROUTINE FOR STATE DETERMINATION CALCULATIONS.
C     ELEMENT TYPE = 11. THREE DIMENSIONAL TENDON ELEMENTS.
C     ***************************************************
C
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      DIMENSION Q(NDF),FACAL(1),FACIL(1)
      COMMON /INFGR / NGR,NELS,MFST,IGRHED(10),NINFC,LSTAT,
     *                LSTF,LSTC,NDOF,DKO,DKT,EPROP(1000)
      COMMON /INFEL / IMEM,KST,LM(18),NODE(6),XX(6),YY(6),
     *                ZZ(6),PROP(1),AREA,XL,B(7),SEL,VTOT,
     *                STOT,TT(3,3),T,SKP(324)
      COMMON /PRECIS/ ZERO,ONE
C
      KST = 0
      RETURN
      END
C-----------------------------------------------------------
      SUBROUTINE RINT11 (NDF,Q,VEL,FE,FD,TIME,FACAL,FACIL,ALFA)
C-----------------------------------------------------------
C     ***************************************************
C     SUBROUTINE FOR INTERNAL FORCE CALCULATIONS,
C     ELEMENT TYPE = 11 THREE DIMENSIONAL TENDON ELEMENTS.
C     ***************************************************
C
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      DIMENSION Q(NDF),VEL(NDF),FE(NDF),FD(NDF),FT(12),
     *          FACAL(1),FACIL(1)
      COMMON /INFGR / NGR,NELS,MFST,IGRHED(10),NINFC,LSTAT,
     *                LSTF,LSTC,NDOF,DKO,DKT,EPROP(1000)
```

```
       COMMON /INFEL / IMEM,KST,LM(18),NODE(6),XX(6),YY(6),
     *                 ZZ(6),PROP(1),AREA,XL,B(7),SEL,VTOT,STOT,
     *                 TT(3,3),T,SKP(324)
       COMMON /STLDPT/ NPNF,NPFF,NPTP,NPP4,NPDP,NETP,NESW,NITSR,
     *                 NSPATT,NSEG
       COMMON /PRECIS/ ZERO,ONE
C
C  CALCULATE THE INTERNAL RESISTING FORCE
C
c       IF(LM(1).EQ.0.and.LM(2).EQ.0.and.LM(3).EQ.0) THEN
        IF(XX(1).EQ.XX(2)) THEN
           NA=4
           NB=5
           NC=6
        ELSE
           NA=1
           NB=2
           NC=3
       ENDIF
       SA=ZERO
       SB=ZERO
       SC=ZERO
C
C    CALCUALTE THE TENDON LOSS DUE TO DEFORMATION
C
       DO 400 K=1,NDOF
          NAK=(NA-1)*NDOF+K
          NBK=(NB-1)*NDOF+K
          NCK=(NC-1)*NDOF+K
          sa=sa+skp(nak)*q(k)
          sb=sb+skp(nbk)*q(k)
          sc=sc+skp(nck)*q(k)
  400     CONTINUE
C
C     LOADING FACTOR
C
      IF(NITSR.EQ.1) THEN
c-----compute factor of tendon force to include -----
      npt=npnf+npff+nptp+npp4+npdp+netp+nesw+nitsr
      npt1=npt-npdp
      fac=facal(npt1)+alfa*facil(npt)
      else
      fac=ZERO
      endif
C
      DSL=SQRT(SA*SA+SB*SB+SC*SC)
      if (sa.lt.zero)  dsl=-dsl
      SEL=fac*SEL-dsl
      STOT=SEL/AREA
      VTOT=STOT/EPROP(1)
      CALL FORVER(sel,B,FT,XX,YY,ZZ)
C
```

```
C     TRANSFORMATION FROM LOCAL TO GLOBAL
C
      CALL TRAN3(TT,FT,FE,18)
C
C     CALCULATE THE DAMPING FORCE
C
        DO 500 I=1,NDOF
500   FD(I)=ZERO
1000  RETURN
      END
C*************************************************************
C     SUBROUTINE TRAN3                                       *
C*************************************************************
      SUBROUTINE TRAN3(T,FL,F,NDF)
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION T(3,3),F(NDF),FL(12)
      COMMON/PRECIS/ZERO,ONE
      DO 100 I=1,NDF
100   F(I)=ZERO
      DO 200 I=1,3
200   F(I)=T(1,I)*FL(1)
      DO 300 I=16,18
        II=I-15
300   F(I)=T(1,II)*FL(12)
      DO 400 I=4,6
        II=I-3
      DO 400 K=2,3
        KK=K-1
400   F(I)=F(I)+T(KK,II)*FL(K)
      DO 500 I=7,9
        II=I-6
      DO 500 K=4,5
        KK=K-3
500   F(I)=F(I)+T(KK,II)*FL(K)
      DO 600 L=6,9,3
      DO 600 J=L+4,L+6
      DO 600 K=L,L+2
         JJ=J-L-3
         KK=K-L+1
600   F(J)=F(J)+T(KK,JJ)*FL(K)
      DO 700 I=16,18
         II=I-15
700   F(I)=T(1,II)*FL(12)
      RETURN
      END
C*************************************************************
C     SUBROUTINE FORVER ---                                  *
C*************************************************************
      SUBROUTINE FORVER(T1,B,FT,XX,YY,ZZ)
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION B(7),FT(12),XX(6),YY(6),ZZ(6)
      FT(1)=-T1
      FT(2)=T1*B(6)
      FT(3)=T1*B(1)
      FT(4)=-FT(2)
      FT(5)=-FT(3)
      FT(6)=T1*B(7)
      FT(7)=-T1*B(2)
      FT(8)=-T1*B(5)
      FT(9)=-FT(6)
      FT(10)=-FT(7)
      FT(11)=-FT(8)
      FT(12)=T1
      IF (XX(1).EQ.XX(2)) THEN
        FT(1)=0.0
        FT(2)=-T1*B(3)
      ENDIF
      IF (XX(6).EQ.XX(5)) THEN
        FT(9)=T1*B(4)
        FT(12)=0.0
      ENDIF
      RETURN
      END
C---------------------------------------------------------------
      SUBROUTINE INIT11(NJT,NDF,RF,FACIL)
C---------------------------------------------------------------
C     ***************************************************
C     SUBROUTINE TO COMPUTE THE INITIAL FORCE
C     ELEMENT TYPE = 11  THREE DIMENSIONAL TENDON ELEMENTS.
C     **************************************************
C
      IMPLICIT DOUBLE PRECISION(A-H,O-Z)
      COMMON /INFGR / NGR,NELS,MFST,IGRHED(10),NINFC,LSTAT,
     *                LSTF,LSTC,NDOF,DKO,DKT,EPROP(1000)
      COMMON /INFEL / IMEM,KST,LM(18),NODE(6),XX(6),YY(6),
     *                ZZ(6),PROP(1),AREA,XL,B(7),SEL,VTOT,
     *                STOT,TT(3,3),T,SKP(324)
      COMMON /PRECIS/ ZERO,ONE
      COMMON /STLDPT/ NPNF,NPFF,NPTP,NPP4,NPDP,NETP,NESW,NITSR,
     *                NSPATT,NSEG
      DIMENSION RF(NDF),fi(12),FACIL(1)
C
C-----CHECK IF INITIAL FORCE IS TO BE INCLUDED -----
C
      IF(NITSR.EQ.1) THEN
c-----compute factor of tendon force to include -----
      npt=npnf+npff+nptp+npp4+npdp+netp+nesw+nitsr
c-----for facal, npt1 = npt - npdp
c-----fac = facal(npt1) + alfa*facil(npt)
      fac=facil(npt)
C
C     COMPUTE INITIAL FORCE
C
      P=FAC*T
      CALL FORVER(P,B,FI,XX,YY,ZZ)
C
C     TRANSFORMATION FROM LOCAL TO GLOBAL
C
      CALL TRAN3(TT,FI,RF,18)
      ENDIF
      RETURN
      END
C-------------------------------------------------------
      SUBROUTINE OUTS11 (KPR,TIME)
C--------------------------------------------------------
C     ***************************************************
C     SUBROUTINE TO PRINT TIME HISTORY OF CURRENT STATE,
C     ELEMENT TYPE = 11 THREE DIMENSIONAL TENDON ELEMENTS.
C     ***************************************************
C
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      COMMON /TAPES / NIU,NOU,NT1,NT2,NT3,NT4,NT5,NT6,NT7,NT8,
     *                NT9,NT10,NT11,NT12,NT13,NT14,NT15,NT16
      COMMON /INFGR / NGR,NELS,MFST,IGRHED(10),NINFC,LSTAT,
     *                LSTF,LSTC,NDOF,DKO,DKT,EPROP(1000)
      COMMON /INFEL / IMEM,KST,LM(18),NODE(6),XX(6),YY(6),
     *                ZZ(6),PROP(1),AREA,XL,B(7),SEL,VTOT,STOT,
     *                TT(3,3),T,SKP(324)
C
      IF (IMEM.EQ.MFST) KHED = 0
      FTOT=SEL
      IF (KPR.EQ.0) RETURN
C
      IF (KHED.NE.0) GO TO 100
C
      KHED = 1
      KKPR = IABS(KPR)
      WRITE(NOU,2000) KKPR,TIME,IGRHED
C
100   WRITE(NOU,2010) IMEM,NODE(1),NODE(6),FTOT,STOT,VTOT
C
2000  FORMAT (///18H RESULTS FOR GROUP,I3,
     *  44H (THREE DIMENSIONAL TENDON ELEMENTS), TIME = E11.4//
     *  5X,10A4//
     *  5X,4HELEM,5X,4HNODE,5X,4HNODE,8X,6HTENDON,8X,6HTENDON,
     *  5X,9H TENDON  /
     *  5X,4H NO.,5X,4H   I,5X,4H   J,4X,8X,6HFORCE ,8X,6HSTRESS,
     *  3X,11HDEFORMATION /)
2010  FORMAT (3I9,3E14.4)
C
      RETURN
      END
```

APPENDIX C

LISTINGS OF THE TENDON ELEMENT WITH FRICTION

```
C-----------------------------------------------------------
      SUBROUTINE INEL11 (NJT,NDKOD,X,Y,Z,KEXEC)
C-----------------------------------------------------------
C     ***************************************************
C     SUBROUTINE TO READ, GENERATE AND PRINT ELEMENT DATA.
C     ELEMENT TYPE = 11 THREE DIMENSIONAL TENDON ELEMENTS.
C     ***************************************************
C
      IMPLICIT DOUBLE PRECISION(A-H,O-Z)
      DIMENSION NDKOD(NJT,3),X(NJT),Y(NJT),Z(NJT),ICOM(882)
      COMMON /TAPES / NIU,NOU,NT1,NT2,NT3,NT4,NT5,NT6,NT7,NT8,
     *                NT9,NT10,NT11,NT12,NT13,NT14,NT15,NT16
      COMMON /INFGR / NGR,NELS,MFST,IGRHED(10),NINFC,LSTAT,
     *                LSTF,LSTC,NDOF,DKO,DKT,EPROP(5,200)
      COMMON /INFEL / IMEM,KST,LM(18),NODE(6),INS(6),INSO(6),
     *      XX(6), YY(6),ZZ(6),PROP(5),AREA,XL(5),B(7),SEL(5),
     *      STOT(5),TT(3,3),T,TI(5),QL(12),SHA1,SHA2,
     *      BK(4,4),SK(5),IP,IDUM,ANSL,SKP(324)
      COMMON /PRECIS/ ZERO,ONE
      COMMON /FRICT /F(5),AK(4,4),SF(4,4),R(4),SS(4,4),RR(4,4)
      COMMON /WORK  / NODC(6),XE(6),YE(6),ZE(6)
      EQUIVALENCE (IMEM,ICOM(1))
C
      DIMENSION IAST(3)
      DATA IAST /2H  ,2H *,2H**/
C
C     ----------------------------------------------------
C     MEANINGS OF VARIABLES IN COMMON BLOCK /INFEL/
C
C     IMEM      - ELEMENT NUMBER.
C     KST       - STIFFNESS UPDATE CODE.
C     LM(18)    - LOCATION MATRIX.
C     NODE(6)   - NODE NUMBERS FROM END I TO J.
C     INS(6)    - CODE OF NODE'S SLIDING.
C     INSO(6)   - PREVIOUS CODE OF NODE'S SLIDING.
C     PROP(5)   - MATERIAL PROPERTIES,EA,fs,fd,ds,dd.
C     AREA      - CROSS SECTIONAL AREA.
C     XL(5)     - ELEMENT LENGTH OF EACH SEGMENT.
C     XX(6)     - LOCAL COORDINATES OF NODE I TO J.
C     B(8)      - DIRECTION PARAMETERS.
C     TT(3,3)   - TRANSFORMATION MATRIX.
C     STOT(5)   - STRESS OF EACH SEGMENT.
C     SEL(5)    - AXIAL FORCE OF EACH SEGMENT.
C     SKP(324) - CURRENT TANGENT OR EFFECTIVE STIFFNESS MATRIX.
C     -----------------------------------------------------
C
C     ELEMENT GROUP PARAMETERS
C
      IF (MFST.LE.0) MFST = 1
C
      NDOF  = 18
      LSTAT = 234
      LSTF  = 648
      LSTC  = 0
      NINFC = 882
      KST   = 1
C
      DO 100 I=3,882
100   ICOM(I) = 0
C
C     PRINT ELEMENT GROUP INFORMATION
C
      WRITE(NOU,2000) NGR,IGRHED,NELS,MFST,NDOF,NINFC,DKO,DKT
C
C     READ AND PRINT MATERIAL PROPERTIES & ANCHORAGE SLIP
C     LENGTH
      READ (NIU,1000) NMAT
      IF (NMAT.LE.0) NMAT = 1
C
      WRITE(NOU,2010) NMAT
      DO 110 I=1,NMAT
110   READ (NIU,1005) M,(EPROP(J,M),J=1,5)
      WRITE(NOU,2020) (M,(EPROP(J,M),J=1,5),M=1,NMAT)
      READ (NIU,1008) NT,IP,ANSL
      WRITE(NOU,2025) NT,IP,ANSL
      WRITE(NOU,2026)
C
C     READ, GENERATE AND PRINT ELEMENT DATA
C
      IMEM  = MFST
      NLAST = MFST + NELS - 1
C
140   READ (NIU,1010) N,NODC,MT,AO,FORC,ND
C
      IF (MT.LE.0) MT = 1
      IF (ND.EQ.0) ND = 1
C
150   IF (N - IMEM) 155,160,210
C
155   JFAST  = IAST(3)
      KEROR = 1
      WRITE(NOU,2040) JFAST,N,NODC,MT,AO,FORC
      GO TO 290
C
160   DO 170 I=1,6
170   NODE(I) = NODC(I)
C
      KEROR = 0
      NDIF  = ND
      MTYP  = MT
      AREA  = AO
         T  = FORC
      JFAST = IAST(1)
      IF (N.NE.NLAST) GO TO 140
      GO TO 210
C
```

```
190   DO 200 I=1,6
200   NODE(I) = NODE(I) + NDIF
      JFAST = IAST(2)
C
210   WRITE(NOU,2040)JFAST,IMEM,NODE,MTYP,AREA,T
C
C     ELEMENT CONNECTIVITY AND DIRECTION
C     TRANSFORMATION OF COORDINATES
C
      DO 220 I=1,6
      NOD = NODE(I)
      II = 3*I
      CALL NCODLM(LM(II-2),NJT,NOD,1)
      CALL NCODLM(LM(II-1),NJT,NOD,2)
      CALL NCODLM(LM(II),NJT,NOD,3)
      XE(I)=X(NOD)
      YE(I)=Y(NOD)
220   ZE(I)=Z(NOD)
      CALL TRAN1(XE,YE,ZE,XX,YY,ZZ,TT)
C
      Z4=ZZ(5)-ZZ(4)
      DZ4=DABS(Z4)
      DX1=XX(2)-XX(1)
      IF(XX(1).EQ.XX(2)) DX1=ZERO
      DX2=XX(3)-XX(2)
      DX3=XX(4)-XX(3)
      DX4=XX(5)-XX(4)
      DX5=XX(6)-XX(5)
      IF(XX(6).EQ.XX(5)) DX5=ZERO
      DY2=YY(2)-YY(3)
      DY4=YY(5)-YY(4)
      XL(1)=DX1
      XL(2)=SQRT(DX2*DX2+DY2*DY2)
      XL(3)=DX3
      XL(4)=SQRT(DZ4*DZ4+DX4*DX4+DY4*DY4)
      XL(5)=DX5
      CY1=DY2/XL(2)
      CY2=DY4/XL(4)
      SY1=DX2/XL(2)
      CX2=DX4/XL(4)
      CZ2=Z4/XL(4)
      SX11=1.-SY1
      CX22=1.-CX2
C
C     INITIALIZE DIRECTION PARAMETERS
C
C
      B(1)=CY1
      B(2)=CY2
      B(3)=SY1
      B(4)=CX2
      B(5)=CZ2
      B(6)=SX11
      B(7)=CX22
      PROP(1)=EPROP(1,MT)*AREA
      PROP(2)=EPROP(2,MT)
      PROP(3)=EPROP(3,MT)
      PROP(4)=EPROP(4,MT)
      PROP(5)=EPROP(5,MT)
C
C     INITIALIZE INS,INSO,IST
C
      DO 250 I=1,6
      INS(I)=0
      INSO(I)=0
250   CONTINUE
C
C     INITIALIZE QL
C
      DO 260 I=1,12
260   QL(I)=ZERO
C
C     COMPUTE THE FRICTION LOSS DURING TENSIONING
C
      ANG1=DACOS(B(3))
      ANG2=DACOS(B(4))
      SHA1=DSIN(ANG1*0.5D0)
      SHA2=DSIN(ANG2*0.5D0)
      write(*,*)'ang1,2',ang1,ang2
C     if the tendon is not symmetrical then stop
      IF(NT.NE.2) go to 265
      DIV=DABS(ANG1-ANG2)/DMIN1(ANG1,ANG2)
      IF(DIV.GT.1.E-5) THEN
         KEROR=2
         GO TO 290
      ENDIF
C
265   IF(XL(1).EQ.ZERO) THEN
        TI(1)=ZERO
        TI(2)=T
        IB=3
      ELSE
        TI(1)=T
        IB=2
      ENDIF
C
      IF(XL(5).EQ.ZERO) THEN
        TI(5)=ZERO
        IE=4
      ELSE
        IE=5
      ENDIF
C     for tensioning at both ends
      IF(NT.EQ.2) IE=3
      DO 310 I=IB,IE
```

```
          IF(I.LT.4) THEN
            COFF=PROP(3)*SHA1
          ELSE
            COFF=PROP(3)*SHA2
          ENDIF
            F(I)=2.*TI(I-1)*COFF/(1.+COFF)
            TI(I)=TI(I-1)-F(I)
310      CONTINUE
C        for tensioning at two ends
        IF(NT.EQ.1) go to 311
        TI(4)=TI(2)
        TI(5)=TI(1)
C
C       PRINT THE INITIAL FORCE AFTER THE FRICTION LOSS
C
311     WRITE(NOU,3000)
        WRITE(NOU,3050)IMEM,NODE(1),NODE(6)
        DO 320 I=1,5
320      WRITE(NOU,3100) NODE(I),NODE(I+1),TI(I)
C
C       EVALUATE THE LOSS DUE TO ANCHORAGE SLIP
C       CALCULATE THE STIFFNESS OF EACH SEGMENT
C
        DO 350 I=1,5
        IF(XL(I).EQ.ZERO) THEN
          SK(I)=ZERO
        else
          SK(I)=PROP(1)/XL(I)
        ENDIF
350      CONTINUE
c
        IF(PROP(2).EQ.ZERO) GO TO 410
C
C       INITIALIZE THE TENSION LOSS
C
        IF(XL(1).NE.ZERO) THEN
          TLOSS=ANSL*SK(1)
          TI(1)=TI(1)-TLOSS
        ELSE
          TLOSS=ANSL*SK(2)
          TI(2)=TI(2)-TLOSS
        ENDIF
C
C
C       FORM THE SLIP STIFFNESS MATIX (W/O FRICTION)
C
        CALL BSTIFF(SK,BK)
C
C       FORM THE FRICTION TERMS
C       ASSUME THE TENDON IS TENSIONED AT NODE I---
C       ------for one tensioning end
C
        COFF1=PROP(3)*SHA1
```

```
       COFF2=PROP(3)*SHA2
       AK(1,1)=coff1*(SK(1)-SK(2))
       AK(1,2)=coff1*SK(2)
       AK(2,1)=-coff1*SK(2)
       AK(2,2)=coff1*(SK(2)-SK(3))
       AK(2,3)=coff1*SK(3)
       AK(3,2)=-coff2*SK(3)
       AK(3,3)=coff2*(SK(3)-SK(4))
       AK(3,4)=coff2*SK(4)
       AK(4,3)=-coff2*SK(4)
       AK(4,4)=coff2*(SK(4)-SK(5))
C      for tensioning at two ends
       If(nt.eq.1) go to 360
       ak(3,2)=-ak(3,2)
       ak(3,3)=-ak(3,3)
       ak(3,4)=-ak(3,4)
       ak(4,3)=-ak(4,3)
       ak(4,4)=-ak(4,4)
C
C      FORM THE TOTAL SLIP STIFFNESS MATRIX
C
360    CALL TSTIFF(AK,BK,SF)
c
c      check node II if the loss of anchorage slip affects
c      ti(2)
c
       ns=0
       pro=PROP(2)/PROP(3)
       if(xl(1).eq.ZERO) go to 500
       call zeror(r)
       ff1=(ti(1)+ti(2))*coff1*pro
       df1=ti(2)-ti(1)
       if(df1.le.ff1) go to 700
       ns=1
       R(1)=df1-ff1/pro
       call sol(r,sf,ns,2,ss,rr)
       call force(ti,sk,r,2)
c
c      check node III if the loss of anchorage slip affects
c      ti(3)
c
500    call zeror(r)
       ff1=(ti(2)+ti(3))*coff1*pro
       df1=ti(3)-ti(2)
       if(df1.le.ff1) go to 700
C      for tensioning at two ends
       if(nt.eq.2) then
        if(xl(1).eq.zero) then
         ns=2
         idd=4
        else
         ns=4
         idd=5
        endif
         ti(5)=ti(1)
         ti(4)=ti(2)
         r(2)=dfl-ffl/pro
         r(3)=-r(2)
         call sol(r,sf,ns,idd,ss,rr)
         call force(ti,sk,r,5)
         go to 710
      endif
      ns=ns+1
      R(2)=dfl-ffl/pro
      call sol(r,sf,ns,3,ss,rr)
      call force(ti,sk,r,3)
c
c     check node JJJ if the loss of anchorage slip affects
c     ti(4)
c
      call zeror(r)
      ffl=(ti(3)+ti(4))*coffl*pro
      dfl=ti(4)-ti(3)
      if(dfl.le.ffl) go to 700
      ns=ns+1
      R(3)=dfl-ffl/pro
      call sol(r,sf,ns,4,ss,rr)
      call force(ti,sk,r,4)
c
c     check node JJ if the loss of anchorage slip affects
c     ti(5)
c
      if(xl(5).eq.ZERO) go to 700
      call zeror(r)
      ffl=(ti(4)+ti(5))*coffl*pro
      dfl=ti(5)-ti(4)
      if(dfl.le.ffl) go to 500
      ns=ns+1
      R(4)=dfl-ffl/pro
      call sol(r,sf,ns,5,ss,rr)
      call force(ti,sk,r,5)
C
C     PRINT THE INITIAL FORCE AFTER ANCHORAGE SLIP
C
700   if(nt.eq.2) then
         ti(4)=ti(2)
         ti(5)=ti(1)
      endif
710   WRITE(NOU,3200)
      WRITE(NOU,3050)IMEM,NODE(1),NODE(6)
      DO 400 I=1,5
400   WRITE(NOU,3100) NODE(I),NODE(I+1),TI(I)
C
C     COMPUTE STIFFNESS MATRIX PROFILE AND TRANSFER ELEMENT
C     DATA TO TAPE
C
```

```
 410   CALL BAND(LM,NDOF,NDKOD)
       CALL COMPAC
 C
 C     CHECK LAST ELEMENT
 C
       IF (IMEM.EQ.NLAST) GO TO 290
       IMEM = IMEM + 1
       IF (IMEM.EQ.N) GO TO 160
       GO TO 190
 C
 C     ERROR AND ELEMENT DATA GENERATION MESSAGES
 C
 290   IF (KEROR.EQ.0) GO TO 300
 C
       IF(KEROR.EQ.2) THEN
          WRITE(NOU,2061)
          CALL EXIT
       ENDIF
       WRITE(NOU,2060)
       CALL EXIT
 300   RETURN
 1000  FORMAT (I5)
 1005  FORMAT (I5,5F10.0)
 1008  FORMAT (2I5,F10.0)
 1010  FORMAT (8I5,2F10.0,1X,I5)
 2000  FORMAT (26H ELEMENT GROUP INDICATOR = I3,
     *    36H (THREE DIMENSIONAL TENDON ELEMENTS)//5X,10A4//
     *    5X,43HNUMBER OF ELEMENTS IN THIS GROUP ........ = I5/
     *    5X,43HNUMBER OF FIRST ELEMENT IN THIS GROUP ... = I5//
     *    5X,43HNUMBER OF DEGREES OF FREEDOM PER ELEMENT  = I5/
     *    5X,43HLENGTH OF ELEMENT INFORMATION ARRAY ..... = I5//
     *    5X,43HDAMPING COEFFICIENT, BETA-O ............. = F11.5/
     *    5X,43HDAMPING COEFFICIENT, BETA-T ............. = F11.5)
 2010  FORMAT (//20H MATERIAL PROPERTIES//
     *         5X,36HNUMBER OF DIFFERENT MATERIAL TYPES = I5//)
 2020  FORMAT (5X,38HMATERIAL NUMBER ................... = I9/
     *    5X,43HMODULUS OF THE TENDON ................. = E12.5/
     *    5X,43HSTA. COFF. OF FRIC. BETWEEN DUCT & TENDON = E12.5/
     *    5X,43HDYN. COFF. OF FRIC. BETWEEN DUCT & TENDON = E12.5/
     *    5X,43HSTA. COFF. OF FRIC. BETWEEN CON. & TENDON = E12.5/
     *    5X,43HDYN. COFF. OF FRIC. BETWEEN CON. & TENDON = E12.5)
 2025  FORMAT (//40H TENSIONING METHOD & ANCHORAGE PROPERTY //
     *    5X,43HNUMBER OF TENSIONING ENDS .............  = I5/
     *    5X,43HPRESTRESSING METHODS ..................  = I5/
     *    5X,43HANCHORAGE SLIP LENGTH FOR EACH TENDON ... = F12.5)
 2026  FORMAT (//20H ELEMENT INFORMATION//)
 2040  FORMAT (/A2/
     *    5X,43HELEMENT NUMBER ......................... = I6//
     *    5X,10HNODE  I  =I8,5X,10HNODE II  =I8,5X,10HNODE III =I8/
     *    5X,10HNODE JJJ =I8,5X,10HNODE JJ  =I8,5X,10HNODE  J  =I8/
     *    5X,13HMATERIAL NO =I7,5X,8H AREA  =F12.4/
     *    5X,15HINITIAL FORCE =F12.4/)
 2050  FORMAT (1X,A2,I6,7I9,2F12.4)
```

```
2060  FORMAT (///15H ERROR MESSAGES//
     * 5X,3HKEY,5X,29H------------- MESSAGE -------,5X,5HERROR//
     * 5X,3H   ,5X,29HELEMENT DATA INPUT DIRECTLY  ,5X,5HNONE /
     * 5X,3H  *,5X,29HELEMENT DATA GENERATED       ,5X,5HNONE /
     * 5X,3H **,5X,29HELEMENT CARD OUT OF SEQUENCE ,5X,5HFATAL/)
2061  FORMAT (///15H ERROR MESSAGES//
     * 5X,43H ----TENDON IS NOT SYMMETRICAL-------------/)
3000  FORMAT(//33HINITIAL FORCE AFTER FRICTION LOSS//)
3050  FORMAT(5X,8HELEM NO.,I5,5X,5HNODE1,I5,5X,5HNODE6,I5//
     *       5X,4HNODE,1X,3H---,1X,4HNODE,5X,10HTEN. FORCE/)
3100  FORMAT(2I9,E15.4)
3200  FORMAT(//34HINITIAL FORCE AFTER ANCHORAGE SLIP//)
      END
C**********************************************************
C       SUBROUTINE TRAN1-TRANSFORMATION of COORDINATES      *
C**********************************************************
      SUBROUTINE TRAN1(X,Y,Z,XX,YY,ZZ,T)
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION X(6),Y(6),Z(6),T(3,3),XX(6),YY(6)
     *          ,ZZ(6),V(4,4)
      CALL VECTOR(V(1,1),X(3),Y(3),Z(3),X(4),Y(4),
     * Z(4))
      CALL VECTOR(V(1,4),X(3),Y(3),Z(3),X(1),Y(1),Z(1))
      CALL CROS(V(1,1),V(1,4),V(1,3))
      CALL CROS(V(1,3),V(1,1),V(1,2))
         DO 100 K=1,3
         DO 100 L=1,3
  100 T(K,L)=V(L,K)
         DO 200 N=1,6
      XX(N)=T(1,1)*X(N)+T(1,2)*Y(N)+T(1,3)*Z(N)
      YY(N)=T(2,1)*X(N)+T(2,2)*Y(N)+T(2,3)*Z(N)
      ZZ(N)=T(3,1)*X(N)+T(3,2)*Y(N)+T(3,3)*Z(N)
  200    CONTINUE
      RETURN
      END
C---------------------------------------------- VECTOR
      SUBROUTINE VECTOR (V,XI,YI,ZI,XJ,YJ,ZJ)
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION V(4)
      X=XJ-XI
      Y=YJ-YI
      Z=ZJ-ZI
      V(4)=SQRT(X*X+Y*Y+Z*Z)
      V(3)=Z/V(4)
      V(2)=Y/V(4)
      V(1)=X/V(4)
      RETURN
      END
C---------------------------------------------- CROSS
      SUBROUTINE CROS(A,B,C)
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION A(4),B(4),C(4)
      COMMON /IOLIST/ NTM,NTR,NIN,NOT,NT1,NFL,NT2,NT3,NT4,NT5
```

```
C-----------------------------------------------------
      X=A(2)*B(3)-A(3)*B(2)
      Y=A(3)*B(1)-A(1)*B(3)
      Z=A(1)*B(2)-A(2)*B(1)
      XYZ = X*X + Y*Y + Z*Z
      IF(XYZ.GT.0.0) GO TO 100
      WRITE (NTM,2000)
      WRITE (NOT,2000)
      STOP
  100 C(4)=SQRT(XYZ)
      C(3)=Z/C(4)
      C(2)=Y/C(4)
      C(1)=X/C(4)
      RETURN
C
 2000 FORMAT (' ** K NODE IS ALONG ELEMENT SEE MANUAL **'/)
      END
C-----------------------------------------------------BSTIFF
      SUBROUTINE BSTIFF(SK,BK)
      IMPLICIT DOUBLE PRECISION(A-H,O-Z)
      COMMON /PRECIS/ ZERO,ONE
      DIMENSION SK(5),BK(4,4)
      DO 150 J=1,4
      DO 150 I=1,4
150   BK(I,J)=ZERO
      DO 170 I=1,4
170   BK(I,I)=SK(I)+SK(I+1)
      BK(1,2)=-SK(2)
      BK(2,1)=BK(1,2)
      BK(2,3)=-SK(3)
      BK(3,2)=BK(2,3)
      BK(3,4)=-SK(4)
      BK(4,3)=BK(3,4)
      RETURN
      END
C-----------------------------------------------------TSTIFF
      SUBROUTINE TSTIFF(AK,BK,SF)
      IMPLICIT DOUBLE PRECISION(A-H,O-Z)
      DIMENSION AK(4,4),BK(4,4),SF(4,4)
C
C     TOTAL STIFFNESS
C
      DO 5000 J=1,4
      DO 5000 I=1,4
5000  SF(I,J)=BK(I,J)+AK(I,J)
      RETURN
      END
C-----------------------------------------------------ZEROR
      SUBROUTINE ZEROR(R)
      IMPLICIT DOUBLE PRECISION(A-H,O-Z)
      COMMON /PRECIS/ ZERO,ONE
      dimension r(4)
      do 100 i=1,4
```

```
100   r(i)=ZERO
      return
      end
C---------------------------------------------------------FORCE
C
C     CALCULATE THE TENDON FORCES
C
      SUBROUTINE FORCE(sel,sk,R,ind)
      IMPLICIT DOUBLE PRECISION(A-H,O-Z)
      dimension sel(5),sk(5),r(4)
      in=ind
      SEL(1)=SEL(1)+SK(1)*R(1)
      if(ind.eq.5) in=4
      DO 300 I=2,in
        I1=I-1
300   SEL(I)=SEL(I)+SK(I)*(R(I)-R(I1))
      if(ind.le.4) return
      SEL(5)=SEL(5)-SK(5)*R(4)
      RETURN
      END
C--------------------------------------------------------------
      SUBROUTINE STIF11(ISTEP,NDF,CDKO,CDKT,FK,INDFK,ISTFC)
C--------------------------------------------------------------
C     ***********************************************************
C     SUBROUTINE TO COMPUTE ELEMENT TANGENT STIFFNESS MATRIX.
C     ELEMENT TYPE = 11 THREE DIMENSIONAL TENDON  ELEMENTS.
C     ***********************************************************
C
      IMPLICIT DOUBLE PRECISION(A-H,O-Z)
      DIMENSION FK(NDF,NDF)
      COMMON /INFGR / NGR,NELS,MFST,IGRHED(10),NINFC,LSTAT,
     *                LSTF,LSTC,NDOF,DKO,DKT,EPROP(5,200)
      COMMON /INFEL / IMEM,KST,LM(18),NODE(6),INS(6),INSO(6),
     *       XX(6),YY(6),ZZ(6),PROP(5),AREA,XL(5),B(7),SEL(5),
     *       STOT(5),TT(3,3),T,TI(5),QL(12),SHA1,SHA2,
     *       BK(4,4),SK(5),IP,IDUM,ANSL,SKP(324)
      COMMON /STLDPT/ NPNF,NPFF,NPTP,NPP4,NPDP,NETP,NESW,NITSR,
     *                NSPATT,NSEG
      COMMON /WORK  / ST(12,12),SSK(18,18),TS(12,12)
      COMMON /PRECIS/ ZERO,ONE
C-----set to full matrix version -----
      INDFK=0
C
C     LINEAR PART OF EFFECTIVE STIFFNESS
C
C     COMPUTE  STIFFNESS
C
      DO 50 I=1,12
      DO 50 J=1,12
50    ST(I,J)=ZERO
C
C     TURN OFF STIFFNESS AT TRANSFER FOR POST-TENSIONED
C
```

```
C      IF(IP.EQ.0.AND.NSEG.EQ.1) GO TO 510
      IF(NSEG.LT.IP) GO TO 510
C
C     CHECK NODE(II)
C
      IF(INS(2).EQ.0) THEN
C     TRUSS(I,II)
      IF(XX(1).EQ.XX(2)) GO TO 100
      TPR=PROP(1)/XL(1)
      ST(1,1)=TPR
      ST(1,2)=-ST(1,1)
      ST(2,2)=ST(1,1)
        GO TO 100
      ENDIF
C     INS(2)=1 CHECK NODE(III)
        GO TO 150
C
C     CHECK NODE(III)
C
100   IF(INS(3).EQ.0) THEN
C       TRUSS(II,III)
      TPR=PROP(1)/XL(2)
      ST(2,2)=ST(2,2)+TPR*B(3)*B(3)
      ST(2,3)=-TPR*B(1)*B(3)
      ST(2,4)=-TPR*B(3)*B(3)
      ST(2,5)=-ST(2,3)
      ST(3,3)=TPR*B(1)*B(1)
      ST(3,4)=ST(2,5)
      ST(3,5)=-ST(3,3)
      ST(4,4)=-ST(2,4)
      ST(4,5)=ST(2,3)
      ST(5,5)=ST(3,3)
        GO TO 200
      ENDIF
        GO TO 210
150   IF(INS(3).EQ.0) THEN
C     TENDON(I-III)
      TPR=PROP(1)/(XL(1)+XL(2))
      ST(1,1)=TPR
      ST(1,2)=-TPR*B(6)
      ST(1,3)=-TPR*B(1)
      ST(1,4)=-TPR*B(3)
      ST(1,5)=-ST(1,3)
      ST(2,2)=TPR*B(6)*B(6)
      ST(2,3)=TPR*B(6)*B(1)
      ST(2,4)=TPR*B(6)*B(3)
      ST(2,5)=-ST(2,3)
      ST(3,3)=TPR*B(1)*B(1)
      ST(3,4)=TPR*B(1)*B(3)
      ST(3,5)=-ST(3,3)
      ST(4,4)=TPR*B(3)*B(3)
      ST(4,5)=-ST(3,4)
      ST(5,5)=-ST(3,5)
```

```
          GO TO 200
        ENDIF
          GO TO 230
C       CHECK NODE(JJJ)
200     IF(INS(4).EQ.0) THEN
C-----TRUSS(III-JJJ)
        TPR=PROP(1)/XL(3)
        ST(4,4)=ST(4,4)+TPR
        ST(4,6)=-TPR
        ST(6,6)=TPR
          GO TO 300
        ENDIF
           GO TO 310
210     IF(INS(4).EQ.0) THEN
C-----TENDON(II-JJJ)
        TPR=PROP(1)/(XL(2)+XL(3))
        ST(2,2)=ST(2,2)+TPR*B(3)*B(3)
        ST(2,3)=-TPR*B(1)*B(3)
        ST(2,4)=TPR*B(3)*B(6)
        ST(2,5)=-ST(2,3)
        ST(2,6)=-TPR*B(3)
        ST(3,3)=TPR*B(1)*B(1)
        ST(3,4)=-TPR*B(1)*B(6)
        ST(3,5)=-ST(3,3)
        ST(3,6)=TPR*B(1)
        ST(4,4)=TPR*B(6)*B(6)
        ST(4,5)=-ST(3,4)
        ST(4,6)=-TPR*B(6)
        ST(5,5)=ST(3,3)
        ST(5,6)=-ST(3,6)
        ST(6,6)=TPR
          GO TO 300
        ENDIF
          GO TO 330
230     IF(INS(4).EQ.0) THEN
C-----TENDON(I-JJJ)
        TPR=PROP(1)/(XL(1)+XL(2)+XL(3))
        ST(1,1)=TPR
        ST(1,2)=-TPR*B(6)
        ST(1,3)=-TPR*B(1)
        ST(1,4)=-ST(1,2)
        ST(1,5)=-ST(1,3)
        ST(1,6)=-ST(1,1)
        ST(2,2)=TPR*B(6)*B(6)
        ST(2,3)=TPR*B(6)*B(1)
        ST(2,4)=-ST(2,2)
        ST(2,5)=-ST(2,3)
        ST(2,6)=-ST(1,2)
        ST(3,3)=TPR*B(1)*B(1)
        ST(3,4)=-ST(2,3)
        ST(3,5)=-ST(3,3)
        ST(3,6)=-ST(1,3)
        ST(4,4)=-ST(2,4)
```

```
      ST(4,5)=-ST(2,5)
      ST(4,6)=-ST(2,6)
      ST(5,5)=-ST(3,5)
      ST(5,6)=-ST(3,6)
      ST(6,6)=-ST(1,6)
        GO TO 300
      ENDIF
        GO TO 370
C-----CHECK NODE(JJ)
300   IF(INS(5).EQ.0) THEN
C-----TRUSS(JJJ,JJ) TRUSS(JJ,J)
      CALL TRU46(ST,PROP(1),XL,B,XX)
      GO TO 400
      ENDIF
C-----TENDON(JJJ,J)
      CALL TEN46(ST,PROP(1),XL,B)
         GO TO 400
310   IF(INS(5).EQ.0) THEN
C-----TENDON(III,JJ) + TRUSS(JJ,J)
      CALL TT36(ST,PROP(1),XL,B,XX)
      GO TO 400
      ENDIF
C-----TENDON(III,J)
      CALL TEN36(ST,PROP(1),XL,B)
         GO TO 400
330   IF(INS(5).EQ.0) THEN
C-----TENDON(II,JJ) + TRUSS(JJ,J)
      TPR=PROP(1)/(XL(2)+XL(3)+XL(4))
      CALL TEN16(INS,TS,TPR,B)
      DO 335 I=1,12
      DO 335 J=I,12
335   ST(I,J)=ST(I,J)+TS(I,J)
      TPR=PROP(1)/XL(5)
      IF(XX(5).EQ.XX(6)) GO TO 400
      ST(9,9)=ST(9,9)+TPR
      ST(9,12)=-TPR
      ST(12,12)=TPR
         GO TO 400
      ENDIF
C-----TENDON(II-J)
      TPR=PROP(1)/(XL(2)+XL(3)+XL(4)+XL(5))
      CALL TEN16(INS,TS,TPR,B)
      DO 338 I=1,12
      DO 338 J=I,12
338   ST(I,J)=ST(I,J)+TS(I,J)
         GO TO 400
370   IF(INS(5).EQ.0) THEN
C-----TENDON(I,JJ)
      TPR=PROP(1)/(XL(1)+XL(2)+XL(3)+XL(4))
      CALL TEN16(INS,ST,TPR,B)
      TPR=PROP(1)/XL(5)
      IF(XX(5).EQ.XX(6)) GO TO 400
      ST(9,9)=ST(9,9)+TPR
      ST(9,12)=-TPR
      ST(12,12)=TPR
        GO TO 400
      ENDIF
C-----TENDON(I,J)
      TPR=PROP(1)/(XL(1)+XL(2)+XL(3)+XL(4)+XL(5))
      CALL TEN16(INS,ST,TPR,B)
400   DO 500 I=1,12
      DO 500 J=I,12
      ST(J,I)=ST(I,J)
500   CONTINUE
C-----TRANSFORM THE LOCAL STIFFNESS MATRIX INTO GLOBAL MATRIX
510   CALL TRAN2(TT,ST,SSK,18,1)
      IF(ISTFC.EQ.0) GO TO 550
      IJ =0
      DO 140 I=1,NDOF
c      DO 140 J=I,NDOF
      do 140 j=1,ndof
      IJ        = IJ + 1
      STIF      = SSK(I,J)
      FK (I,J) = STIF
c      FK (J,I) = STIF
140   SKP(IJ)   = STIF
      RETURN
C
C     CHANGE STIFFNESS
C
550   IJ=0
      DO 600 I=1,NDOF
      DO 600 J=1,NDOF
      IJ=IJ+1
      STIF=SSK(I,J)
      STIFD=STIF-SKP(IJ)
      FK(I,J)=STIFD
      SKP(IJ)=STIF
600   CONTINUE
      RETURN
      END
C---------------------------------------------------------TRU46
      SUBROUTINE TRU46(S,PRO,XL,B,XX)
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION S(12,12),XL(5),B(7),XX(6)
      COMMON/PRECIS/ZERO,ONE
C-----TRUSS(JJJ-J)
      TMP=PRO/XL(4)
      S(6,6)=S(6,6)+TMP*B(4)*B(4)
      S(6,7)=TMP*B(4)*B(2)
      S(6,8)=TMP*B(4)*B(5)
      S(6,9)=-TMP*B(4)*B(4)
      S(6,10)=-S(6,7)
      S(6,11)=-S(6,8)
      S(7,7)=TMP*B(2)*B(2)
      S(7,8)=TMP*B(2)*B(5)
      S(7,9)=-S(6,7)
      S(7,10)=-S(7,7)
      S(7,11)=-S(7,8)
      S(8,8)=TMP*B(5)*B(5)
      S(8,9)=S(6,11)
      S(8,10)=-S(7,8)
      S(8,11)=-S(8,8)
      TMP1=PRO/XL(5)
      IF(XX(5).EQ.XX(6)) TMP1=ZERO
      S(9,9)=TMP*B(4)*B(4)+TMP1
      S(9,10)=-S(7,9)
      S(9,11)=-S(6,11)
      S(9,12)=-TMP1
      S(10,10)=S(7,7)
      S(10,11)=S(7,8)
      S(11,11)=S(8,8)
      S(12,12)=TMP1
      RETURN
      END
C-----------------------------------------------------------TEN46
      SUBROUTINE TEN46(S,PRO,XL,B)
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION S(12,12),XL(5),B(7)
C-----TENDON(JJJ,J)
      TMP=PRO/(XL(4)+XL(5))
      S(6,6)=S(6,6)+TMP*B(4)*B(4)
      S(6,7)=TMP*B(4)*B(2)
      S(6,8)=TMP*B(4)*B(5)
      S(6,9)=TMP*B(4)*B(7)
      S(6,10)=-S(6,7)
      S(6,11)=-S(6,8)
      S(6,12)=-TMP*B(4)
      S(7,7)=TMP*B(2)*B(2)
      S(7,8)=TMP*B(2)*B(5)
      S(7,9)=TMP*B(2)*B(7)
      S(7,10)=-S(7,7)
      S(7,11)=-S(7,8)
      S(7,12)=-TMP*B(2)
      S(8,8)=TMP*B(5)*B(5)
      S(8,9)=TMP*B(5)*B(7)
      S(8,10)=-S(7,8)
      S(8,11)=-S(8,8)
      S(8,12)=-TMP*B(5)
      S(9,9)=TMP*B(7)*B(7)
      S(9,10)=-S(7,9)
      S(9,11)=-S(8,9)
      S(9,12)=-TMP*B(7)
      S(10,10)=S(7,7)
      S(10,11)=S(7,8)
      S(10,12)=-S(7,12)
      S(11,11)=-S(8,11)
      S(11,12)=-S(8,12)
      S(12,12)=TMP
      RETURN
      END
C------------------------------------------------------TT36
      SUBROUTINE TT36(S,PRO,XL,B,XX)
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION S(12,12),XL(5),B(7),XX(6)
      COMMON/PRECIS/ZERO,ONE
C-----TENDON(III,JJ) + TRUSS(JJ,J)
      TMP=PRO/(XL(3)+XL(4))
      S(4,4)=S(4,4)+TMP
      S(4,6)=-TMP*B(7)
      S(4,7)=TMP*B(2)
      S(4,8)=TMP*B(5)
      S(4,9)=-TMP*B(4)
      S(4,10)=-S(4,7)
      S(4,11)=-S(4,8)
      S(6,6)=TMP*B(7)*B(7)
      S(6,7)=-TMP*B(7)*B(2)
      S(6,8)=-TMP*B(7)*B(5)
      S(6,9)=TMP*B(7)*B(4)
      S(6,10)=-S(6,7)
      S(6,11)=-S(6,8)
      S(7,7)=TMP*B(2)*B(2)
      S(7,8)=TMP*B(2)*B(5)
      S(7,9)=-TMP*B(2)*B(4)
      S(7,10)=-S(7,7)
      S(7,11)=-S(7,8)
      S(8,8)=TMP*B(5)*B(5)
      S(8,9)=-TMP*B(5)*B(4)
      S(8,10)=-S(7,8)
      S(8,11)=-S(8,8)
      TMP1=PRO/XL(5)
      IF(XX(5).EQ.XX(6)) TMP1=ZERO
      S(9,9)=TMP*B(4)*B(4)+TMP1
      S(9,10)=-S(7,9)
      S(9,11)=-S(8,9)
      S(9,12)=-TMP1
      S(10,10)=-S(7,10)
      S(10,11)=-S(7,11)
      S(11,11)=-S(8,11)
      S(12,12)=-S(9,12)
      RETURN
      END
C------------------------------------------------------TEN36
      SUBROUTINE TEN36(S,PRO,XL,B)
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION S(12,12),XL(5),B(7)
C-----TENDON(III,J)
      TMP=PRO/(XL(3)+XL(4)+XL(5))
      S(4,4)=S(4,4)+TMP
      S(4,6)=-TMP*B(7)
      S(4,7)=TMP*B(2)
      S(4,8)=TMP*B(5)
```

```
       S(4,9)=-S(4,6)
       S(4,10)=-S(4,7)
       S(4,11)=-S(4,8)
       S(4,12)=-TMP
       S(6,6)=TMP*B(7)*B(7)
       S(6,7)=-TMP*B(7)*B(2)
       S(6,8)=-TMP*B(7)*B(5)
       S(6,9)=-S(6,6)
       S(6,10)=-S(6,7)
       S(6,11)=-S(6,8)
       S(6,12)=-S(4,6)
       S(7,7)=TMP*B(2)*B(2)
       S(7,8)=TMP*B(2)*B(5)
       S(7,9)=-S(6,7)
       S(7,10)=-S(7,7)
       S(7,11)=-S(7,8)
       S(7,12)=-S(4,7)
       S(8,8)=TMP*B(5)*B(5)
       S(8,9)=-S(6,8)
       S(8,10)=-S(7,8)
       S(8,11)=-S(8,8)
       S(8,12)=-S(4,8)
       S(9,9)=-S(6,9)
       S(9,10)=-S(6,10)
       S(9,11)=-S(6,11)
       S(9,12)=-S(6,12)
       S(10,10)=-S(7,10)
       S(10,11)=-S(7,11)
       S(10,12)=-S(7,12)
       S(11,11)=-S(8,11)
       S(11,12)=-S(8,12)
       S(12,12)=-S(4,12)
       RETURN
       END
C-----------------------------------------------------------------TEN16
      SUBROUTINE TEN16(ID,S,PRO,B)
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION ID(6),S(12,12),B(7)
      COMMON/PRECIS/ZERO,ONE
C-----TENDON(I,J) OR TENDON(I,JJ) OR TENDON(II,JJ) OR
C     TENDON(II,J)
      DO 10 I=1,12
      DO 10 J=I,12
10    S(I,J)=ZERO
      S(1,1)=ONE
      S(1,2)=-B(6)
      S(1,3)=-B(1)
      S(1,4)=-S(1,2)
      S(1,5)=-S(1,3)
      S(1,6)=-B(7)
      S(1,7)=B(2)
      S(1,8)=B(5)
      S(1,9)=-S(1,6)
      S(1,10)=-S(1,7)
      S(1,11)=-S(1,8)
      S(1,12)=-S(1,1)
      S(2,2)=B(6)*B(6)
      S(2,3)=B(6)*B(1)
      S(2,4)=-S(2,2)
      S(2,5)=-S(2,3)
      S(2,6)=B(6)*B(7)
      S(2,7)=-B(6)*B(2)
      S(2,8)=-B(6)*B(5)
      S(2,9)=-S(2,6)
      S(2,10)=-S(2,7)
      S(2,11)=-S(2,8)
      S(2,12)=B(6)
      S(3,3)=B(1)*B(1)
      S(3,4)=-B(1)*B(6)
      S(3,5)=-S(3,3)
      S(3,6)=B(1)*B(7)
      S(3,7)=-B(1)*B(2)
      S(3,8)=-B(1)*B(5)
      S(3,9)=-S(3,6)
      S(3,10)=-S(3,7)
      S(3,11)=-S(3,8)
      S(3,12)=B(1)
        DO 100 I=4,12
100   S(4,I)=-S(2,I)
        DO 200 I=5,12
200   S(5,I)=-S(3,I)
      S(6,6)=B(7)*B(7)
      S(6,7)=-B(7)*B(2)
      S(6,8)=-B(7)*B(5)
      S(6,9)=-S(6,6)
      S(6,10)=-S(6,7)
      S(6,11)=-S(6,8)
      S(6,12)=B(7)
      S(7,7)=B(2)*B(2)
      S(7,8)=B(2)*B(5)
      S(7,9)=-S(6,7)
      S(7,10)=-S(7,7)
      S(7,11)=-S(7,8)
      S(7,12)=-B(2)
      S(8,8)=B(5)*B(5)
      S(8,9)=-S(6,8)
      S(8,10)=-S(7,8)
      S(8,11)=-S(8,8)
      S(8,12)=-B(5)
        DO 300 I=9,12
300   S(9,I)=-S(6,I)
        DO 400 I=10,12
400   S(10,I)=-S(7,I)
      S(11,11)=-S(8,11)
      S(11,12)=-S(8,12)
      S(12,12)=ONE
```

```
C-----FOR TENDON(II-JJ,J)
      IF(ID(2).EQ.0) THEN
        DO 500 I=1,12
500   S(1,I)=ZERO
      S(2,2)=B(3)*B(3)
      S(2,3)=-B(3)*B(1)
      S(2,4)=B(3)*B(6)
      S(2,5)=-S(2,3)
      S(2,6)=-B(3)*B(7)
      S(2,7)=B(3)*B(2)
      S(2,8)=B(3)*B(5)
      S(2,9)=-S(2,6)
      S(2,10)=-S(2,7)
      S(2,11)=-S(2,8)
      S(2,12)=-B(3)
      ENDIF
C-----FOR TENDON(I,II-JJ)
      IF(ID(5).EQ.0) THEN
         DO 600 I=1,12
600   S(I,12)=ZERO
      S(1,9)=-B(4)
      S(2,9)=B(4)*B(6)
      S(3,9)=B(4)*B(1)
      S(4,9)=-S(2,9)
      S(5,9)=-S(3,9)
      S(6,9)=B(4)*B(7)
      S(7,9)=-B(4)*B(2)
      S(8,9)=-B(4)*B(5)
      S(9,9)=B(4)*B(4)
      S(9,10)=-S(7,9)
      S(9,11)=-S(8,9)
      ENDIF
C-----FOR TENDON(II-JJ)
      IF(ID(2).EQ.0.AND.ID(5).EQ.0) THEN
      S(1,9)=ZERO
      S(2,9)=-B(4)*B(3)
      ENDIF
        DO 700 I=1,12
        DO 700 J=I,12
700   S(I,J)=S(I,J)*PRO
      RETURN
      END
C-------------------------------------------------------------TRAN2
      SUBROUTINE TRAN2(T,SL,S,ND,NL)
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION T(3,3),TG(12,18),SL(12,12),S(18,18),ST(12,18)
      COMMON/PRECIS/ ZERO,ONE
      DO 50 I=1,12
      DO 50 J=1,18
      ST(I,J)=ZERO
50    TG(I,J)=ZERO
      DO 60 I=1,18
      DO 60 J=1,18
```

```
60    S(I,J)=ZERO
C-----FORM THE TRANSFORMATION MATRIX TG
      DO 100 J=1,3
100   TG(1,J)=T(1,J)
      DO 150 I=2,3
      DO 150 J=4,6
       II=I-1
       JJ=J-3
150   TG(I,J)=T(II,JJ)
      DO 200 I=4,5
      DO 200 J=7,9
       II=I-3
       JJ=J-6
200   TG(I,J)=T(II,JJ)
      DO 250 L=6,9,3
      DO 250 I=L,L+2
      DO 250 J=L+4,L+6
       II=I-L+1
       JJ=J-L-3
250   TG(I,J)=T(II,JJ)
      DO 300 J=16,18
       JJ=J-15
300   TG(12,J)=T(1,JJ)
C-----EVALUATION ST=SL*TG
      DO 400 I=1,12
      DO 400 J=1,18
      DO 400 K=1,12
400   ST(I,J)=ST(I,J)+SL(I,K)*TG(K,J)
C---- S=TTG*SL*TG
      DO 500 I=1,18
      DO 500 J=I,18
      DO 500 K=1,12
500   S(I,J)=S(I,J)+TG(K,I)*ST(K,J)
      DO 600 I=1,18
      DO 600 J=I,18
600   S(J,I)=S(I,J)
      RETURN
      END
C--------------------------------------------------------
      SUBROUTINE STAT11 (NDF,Q,TIME,FACAL,FACIL,ALFA)
C--------------------------------------------------------
C     ****************************************************
C     SUBROUTINE FOR STATE DETERMINATION CALCULATIONS.
C     ELEMENT TYPE = 11. THREE DIMENSIONAL TENDON ELEMENTS.
C     *****************************************************
C
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      DIMENSION Q(NDF),FACAL(1),FACIL(1),IN(5)
      COMMON /INFGR / NGR,NELS,MFST,IGRHED(10),NINFC,LSTAT,
     *                LSTF,LSTC,NDOF,DKO,DKT,EPROP(5,200)
      COMMON /INFEL / IMEM,KST,LM(18),NODE(6),INS(6),INSO(6),
     *      XX(6),YY(6),ZZ(6),PROP(5),AREA,XL(5),B(7),SEL(5),
     *      STOT(5),TT(3,3),T,TI(5),QL(12),SHA1,SHA2,
     *        BK(4,4),SK(5),IP,IDUM,ANSL,SKP(324)
      COMMON /PRECIS/ ZERO,ONE
      COMMON /PRITC / TOL,WTFAC,RNORM,DENOM,RNUM,WFAC,CONV,
     *                GRACE,NIT,MAXIT,ITKOD,ITPR,RNMN
      COMMON /STLDPT/ NPNF,NPFF,NPTP,NPP4,NPDP,NETP,NESW,NITSR,
     *                NSPATT,NSEG
      COMMON /TRUSS/  TD(5),TRUSF(5)
      COMMON /DIST/   R(4),SF(4,4),SS(4,4),RR(4),AK(4,4)
C
C  CALCULATE THE INTERNAL RESISTING FORCE
C
C     SET INSO(I)=INS
      DO 10 I=1,6
10    INSO(I)=INS(I)
C    TRANSFORM Q TO QL
      CALL TRAN4(TT,Q,QL,NDOF)
C
C    CALCULATE THE AXIAL FORCE AND DEFORMATION OF EACH
C    SEGMENT(TRUSS)
      TD(1)=QL(2)-QL(1)
      TD(2)=(QL(4)-QL(2))*B(3)+(QL(3)-QL(5))*B(1)
      TD(3)=QL(6)-QL(4)
      TD(4)=(QL(9)-QL(6))*B(4)+(QL(10)-QL(7))*B(2)+(QL(11)-
     *      QL(8))*B(5)
      TD(5)=QL(12)-QL(9)
C
C     LOADING FACTOR
C
      IF(NITSR.EQ.1) THEN
c-------compute factor of tendon force to include -----
        npt=npnf+npff+nptp+npp4+npdp+netp+nesw+nitsr
        npt1=npt-npdp
        fac=facal(npt1)+alfa*facil(npt)
        else
        fac=ZERO
      endif
C
C     INITIALIZE SEL
C
      DO 50 I=1,5
      IF(XL(I).EQ.ZERO) THEN
        TRUSF(I)=ZERO
        SEL(I)=ZERO
        else
        TRUSF(I)=fac*TI(I)+SK(I)*TD(I)
        SEL(I)=TRUSF(I)
      ENDIF
50    CONTINUE
      IT=1
C
C     INITIALIZE INS
C
      DO 60 I=1,6
```

```
 60    INS(I)=0
 C
 C     CALCULATE THE DRIVING FORCE & DETERMINE THE NODE'S SLIDING
 C
 70    DO 65 I=2,5
       IN(I)=INS(I)
 65    CONTINUE
       NID=0
       NS=0
       DO 100 I=1,4
       I1=I+1
       IF(XL(I).EQ.ZERO.OR.XL(I1).EQ.ZERO.OR.INS(I1).EQ.1) THEN
         R(I)=ZERO
         ELSE
         IF(I.GE.3) THEN
           SHA=SHA2
           ELSE
           SHA=SHA1
         ENDIF
         DF=SEL(I1)-SEL(I)
         FF=(SEL(I)+SEL(I1))*SHA*PROP(4)
         IF(PROP(4).EQ.ZERO) THEN
           FF1=ZERO
           ELSE
           FF1=FF*PROP(5)/PROP(4)
         ENDIF
         R(I)=DABS(DF)-FF
         IF(R(I).GT.ZERO) THEN
           INS(I1)=1
           R(I)=R(I)-FF1+FF
           NID=1
           IF(DF.LT.ZERO) R(I)=-R(I)
           ELSE
           R(I)=ZERO
         ENDIF
       ENDIF
 100   CONTINUE
       IF(NID.EQ.0) GO TO 350
 C
 C     FOMULATION OF THE STIFFNESS MATRIX OF FORCE DISTRIBUTION
 C
       IF(IT.GT.1) GO TO 180
       CALL BSTIFF(SK,BK)
       DO 150 J=1,4
       DO 150 I=1,4
 150   AK(I,J)=ZERO
 C
 C     FRICTION TERMS
 C
 180   IF(R(1).NE.ZERO) THEN
         AK(1,1)=PROP(5)*(SK(1)-SK(2))*SHA1
         AK(1,2)=PROP(5)*SK(2)*SHA1
         IF(R(1).LT.ZERO) THEN
```

```
            AK(1,1)=-AK(1,1)
            AK(1,2)=-AK(1,2)
          ENDIF
        ENDIF
        IF(R(2).NE.ZERO) THEN
          AK(2,1)=-PROP(5)*SK(2)*SHA1
          AK(2,2)=PROP(5)*(SK(2)-SK(3))*SHA1
          AK(2,3)=PROP(5)*SK(3)*SHA1
          IF(R(2).LT.ZERO) THEN
            AK(2,1)=-AK(2,1)
            AK(2,2)=-AK(2,2)
            AK(2,3)=-AK(2,3)
          ENDIF
        ENDIF
        IF(R(3).NE.ZERO) THEN
          AK(3,2)=-PROP(5)*SK(3)*SHA2
          AK(3,3)=PROP(5)*(SK(3)-SK(4))*SHA2
          AK(3,4)=PROP(5)*SK(4)*SHA2
          IF(R(3).LT.ZERO) THEN
            AK(3,2)=-AK(3,2)
            AK(3,3)=-AK(3,3)
            AK(3,4)=-AK(3,4)
          ENDIF
        ENDIF
        IF(R(4).NE.ZERO) THEN
          AK(4,3)=-PROP(5)*SK(4)*SHA2
          AK(4,4)=PROP(5)*(SK(4)-SK(5))*SHA2
          IF(R(4).LT.ZERO) THEN
            AK(4,3)=-AK(4,3)
            AK(4,4)=-AK(4,4)
          ENDIF
        ENDIF
C
C       TOTAL STIFFNESS
C
        CALL TSTIFF(AK,BK,SF)
C
C       CALCULATE DISPLACEMENTS OF FORCE DISTRIBUTION
C
190     DO 200 I=2,5
        IF(INS(I).EQ.0) THEN
          NS=0
          else
          NS=NS+1
          IF(INS(I+1).EQ.0) THEN
C           IF(NS.EQ.4) THEN
C           CALL CROUT(SF,R,4,1,0)
C           GO TO 200
C           ENDIF
            CALL SOL(R,SF,NS,I,SS,RR)
            ENDIF
        ENDIF
200     CONTINUE
```

```
C
C     CALCULATE THE TENDON FORCES
C
      SEL(1)=SEL(1)+SK(1)*R(1)
      DO 300 I=2,4
        I1=I-1
300   SEL(I)=SEL(I)+SK(I)*(R(I)-R(I1))
      SEL(5)=SEL(5)-SK(5)*R(4)
C
C     CHECK IF THE RECHECKING IS NECESSARY
C
      DO 305 I=2,5
      IF(INS(I).NE.IN(I)) GO TO 310
305   CONTINUE
      GO TO 350
C
C     RECHECK THE UNSLIDING NODES
C
310   DO 320 I=2,5
      IF(INS(I).EQ.0) THEN
        IT=IT+1
        GO TO 70
      ENDIF
320   CONTINUE
C
C     COMPARE INS AND INSO
C
350   KST=0
      DO 600 I=2,5
      IF(INS(I).NE.INSO(I)) KST=1
600   CONTINUE
C
C     TURN ON STIFFNESS
C
C      IF(IP.EQ.0.AND.NSEG.EQ.2) KST=1
      IF(IP.EQ.NSEG.AND.NIT.EQ.1) KST=1
      RETURN
      END
C-----------------------------------------------------TRAN4
      SUBROUTINE TRAN4(T,Q,QL,NDF)
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION T(3,3),Q(NDF),QL(12)
C      DO 100 I=1,14
C100   QL(I)=0.0
      DO 200 I=1,3
200   QL(1)=QL(1)+T(1,I)*Q(I)
      DO 300 I=2,3
      DO 300 K=4,6
        II=I-1
        KK=K-3
300   QL(I)=QL(I)+T(II,KK)*Q(K)
      DO 400 I=4,5
      DO 400 K=7,9
```

```
        II=I-3
        KK=K-6
400   QL(I)=QL(I)+T(II,KK)*Q(K)
      DO 500 I=16,18
        II=I-15
500   QL(12)=QL(12)+T(1,II)*Q(I)
      DO 1500 L=6,9,3
      DO 1500 J=L,L+2
      DO 1500 K=L+4,L+6
         JJ=J-L+1
         KK=K-L-3
1500  QL(J)=QL(J)+T(JJ,KK)*Q(K)
      RETURN
      END
C--------------------------------------------------- SOL
      SUBROUTINE SOL(R,SF,NS,N,SS,RR)
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION R(4),SF(4,4),RR(NS),SS(NS,NS)
      DO 100 I=1,NS
      DO 100 J=1,NS
         II=N-NS-1+I
         JJ=N-NS-1+J
      SS(I,J)=SF(II,JJ)
100   CONTINUE
      DO 200 I=1,NS
         II=N-NS-1+I
      RR(I)=R(II)
200   CONTINUE
      CALL CROUT(SS,RR,NS,1,0)
      DO 300 I=1,NS
         II=N-NS-1+I
      R(II)=RR(I)
300   CONTINUE
      RETURN
      END
C--------------------------------------------------- CROUT
      SUBROUTINE CROUT(A,B,N,LD,KEY)
C-----------------------------------------------------------
C     GENERAL EQUATION SOLVER - M. I. HOIT  1/17/83
C     SYMMETRIC AND NON SYMMETRIC
C
C     KEY = 0   TRIANGULARIZE AND SOLVE
C     KEY = 1   TRIANGULARIZE ONLY
C     KEY = 2   FOWARD AND BACK SUBSTITUTE ONLY
C-----------------------------------------------------------
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION A(N,N),B(N,LD)
C
      IM = 1
      IF(KEY.EQ.2) GO TO 200
      IF(N.EQ.1) GO TO 200
      DO 140 J=2,N
      JM = J - 1
      A(J,1) = A(J,1)/A(1,1)
      IF(J.EQ.2) GO TO 130
      DO 120 I=2,JM
      IM = I - 1
      DO 100 K=1,IM
C-----FORM L(I,J) U(I,J) -----
      A(J,I) = A(J,I) - A(J,K)*A(K,I)
  100 A(I,J) = A(I,J) - A(I,K)*A(K,J)
      D = A(I,I)
C-----CHECK IF ZERO ON DIAGONAL -----
      IF(D.NE.0.0) GO TO 120
      WRITE(NTM,1000) I
      GO TO 140
  120 A(J,I) = A(J,I)/D
  130 DO 135 K=1,JM
  135 A(J,J) = A(J,J) - A(J,K)*A(K,J)
  140 CONTINUE
C-----FOWARD AND BACK SUBSTITUTE ----
  200 IF(KEY.EQ.1) GO TO 900
      DO 400 L=1,LD
C-----FORM Y(I,L) -----
      IF(N.EQ.1) GO TO 300
      DO 210 I=2,N
      IM = I - 1
      DO 210 K=1,IM
  210 B(I,L) = B(I,L) - A(I,K)*B(K,L)
C-----FORM X(I,L) -----
  300 B(N,L) = B(N,L)/A(N,N)
      IF(N.EQ.1) GO TO 400
      IM = N - 1
      DO 330 I=2,N
      JM = IM + 1
      DO 310 J=JM,N
  310 B(IM,L) = B(IM,L) - A(IM,J)*B(J,L)
  320 B(IM,L) = B(IM,L)/A(IM,IM)
  330 IM = IM - 1
C
  400 CONTINUE
  900 RETURN
 1000 FORMAT(' **** ZERO ON THE DIAGONAL OF EQUATION'
     * ' NUMBER ',I5,' ****'/)
      END
C-----------------------------------------------------------------
      SUBROUTINE RINT11 (NDF,Q,VEL,FE,FD,TIME,FACAL,FACIL,ALFA)
C-----------------------------------------------------------------
C     ***********************************************************
C     SUBROUTINE FOR INTERNAL FORCE CALCULATIONS,
C     ELEMENT TYPE = 11 THREE DIMENSIONAL TENDON ELEMENTS.
C     ***********************************************************
C
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      DIMENSION Q(NDF),VEL(NDF),FE(NDF),FD(NDF),FT(12),FACAL(1),
     *          FACIL(1),TTI(5)
```

```
       COMMON /INFGR / NGR,NELS,MFST,IGRHED(10),NINFC,LSTAT,
     *                 LSTF,LSTC,NDOF,DKO,DKT,EPROP(5,200)
       COMMON /INFEL / IMEM,KST,LM(18),NODE(6),INS(6),INSO(6),
     *        XX(6),YY(6),ZZ(6),PROP(5),AREA,XL(5),B(7),SEL(5),
     *        STOT(5),TT(3,3),T,TI(5),QL(12),SHA1,SHA2,
     *        BK(4,4),SK(5),IP,IDUM,ANSL,SKP(324)
       COMMON /STLDPT/ NPNF,NPFF,NPTP,NPP4,NPDP,NETP,NESW,NITSR,
     *                 NSPATT,NSEG
       COMMON /PRECIS/ ZERO,ONE
C
C      CALCULATE STRESS
C
         if(nitsr.eq.1) then
c-------compute factor of tendon force to include -----
         npt=npnf+npff+nptp+npp4+npdp+netp+nesw+nitsr
         npt1=npt-npdp
         fac=facal(npt1)+alfa*facil(npt)
         else
         fac=ZERO
       endif
       do 100 i=1,5
100    tti(i)=fac*ti(i)
350    DO 400 I=1,5
400    STOT(I)=SEL(I)/AREA
       if(nseg.ge.ip) then
       CALL FORVER(SEL,B,FT,XX,YY,ZZ)
        else
       call forver(tti,b,ft,xx,yy,zz)
       endif
C
C      TRANSFORMATION FROM LOCAL TO GLOBAL
C
       CALL TRAN3(TT,FT,FE,18)
C
C   CALCULATE THE DAMPING FORCE
C
       DO 500 I=1,NDOF
500    FD(I)=ZERO
       RETURN
       END
C*****************************************************
C      SUBROUTINE TRAN3                                *
C*****************************************************
       SUBROUTINE TRAN3(T,FL,F,NDF)
       IMPLICIT REAL*8 (A-H,O-Z)
       DIMENSION T(3,3),F(NDF),FL(12)
       COMMON/PRECIS/ZERO,ONE
       DO 100 I=1,NDF
100    F(I)=ZERO
       DO 200 I=1,3
200    F(I)=T(1,I)*FL(1)
       DO 300 I=16,18
         II=I-15
```

```
 300   F(I)=T(1,II)*FL(12)
       DO 400 I=4,6
         II=I-3
       DO 400 K=2,3
         KK=K-1
 400   F(I)=F(I)+T(KK,II)*FL(K)
       DO 500 I=7,9
         II=I-6
       DO 500 K=4,5
         KK=K-3
 500   F(I)=F(I)+T(KK,II)*FL(K)
       DO 600 L=6,9,3
       DO 600 J=L+4,L+6
       DO 600 K=L,L+2
          JJ=J-L-3
          KK=K-L+1
 600   F(J)=F(J)+T(KK,JJ)*FL(K)
       DO 700 I=16,18
          II=I-15
 700   F(I)=T(1,II)*FL(12)
       RETURN
       END
C*********************************************************
C      SUBROUTINE FORVER ---                             *
C*********************************************************
       SUBROUTINE FORVER(SEL,B,FT,XX,YY,ZZ)
       IMPLICIT REAL*8 (A-H,O-Z)
       DIMENSION SEL(5),B(7),FT(12),XX(6),YY(6),ZZ(6)
       FT(1)=-SEL(1)
       FT(2)=SEL(1)-SEL(2)*B(3)
       FT(3)=SEL(2)*B(1)
       FT(4)=SEL(2)*B(3)-SEL(3)
       FT(5)=-FT(3)
       FT(6)=SEL(3)-SEL(4)*B(4)
       FT(7)=-SEL(4)*B(2)
       FT(8)=-SEL(4)*B(5)
       FT(9)=SEL(4)*B(4)-SEL(5)
       FT(10)=-FT(7)
       FT(11)=-FT(8)
       FT(12)=SEL(5)
       IF (XX(1).EQ.XX(2)) THEN
         FT(1)=0.0
         FT(2)=-SEL(2)*B(3)
       ENDIF
       IF (XX(6).EQ.XX(5)) THEN
         FT(9)=SEL(4)*B(4)
         FT(12)=0.0
       ENDIF
       RETURN
       END
C-----------------------------------------------------------
       SUBROUTINE INIT11(NJT,NDF,RF,FACIL)
C-----------------------------------------------------------
```

```
C     ***************************************************
C     SUBROUTINE TO COMPUTE THE INITIAL FORCE
C     ELEMENT TYPE = 11  THREE DIMENSIONAL TENDON ELEMENTS.
C     ************************************************
C
      IMPLICIT DOUBLE PRECISION(A-H,O-Z)
      COMMON /INFGR / NGR,NELS,MFST,IGRHED(10),NINFC,LSTAT,
     *                LSTF,LSTC,NDOF,DKO,DKT,EPROP(5,200)
      COMMON /INFEL / IMEM,KST,LM(18),NODE(6),INS(6),INSO(6),
     *        XX(6),YY(6),ZZ(6),PROP(5),AREA,XL(5),B(7),SEL(5),
     *        STOT(5),TT(3,3),T,TI(5),QL(12),SHA1,SHA2,
     *        BK(4,4),SK(5),IP,IDUM,ANSL,SKP(324)
      COMMON /STLDPT/ NPNF,NPFF,NPTP,NPP4,NPDP,NETP,NESW,NITSR,
     *                NSPATT,NSEG
      COMMON /PRECIS/ ZERO,ONE
      DIMENSION RF(NDF),FI(12),P(5),FACIL(1)
C
C-----CHECK IF INITIAL FORCE IS TO BE INCLUDED -----
C
      IF(NITSR.EQ.1) THEN
c-----compute factor of tendon force to include -----
      npt=npnf+npff+nptp+npp4+npdp+netp+nesw+nitsr
c-----for facal, npt1 = npt - npdp
c-----fac = facal(npt1) + alfa*facil(npt)
      fac=facil(npt)
C
C     CALCULATE THE EQUVALENT INITIAL FORCE
C
      DO 600 I=1,5
600   P(I)=FAC*TI(I)
      CALL FORVER(P,B,FI,XX,YY,ZZ)
C
C     TRANSFORMATION FROM LOCAL TO GLOBAL
C
      CALL TRAN3(TT,FI,RF,18)
      ENDIF
      RETURN
      END
C--------------------------------------------------
      SUBROUTINE OUTS11 (KPR,TIME)
C--------------------------------------------------
C     ***************************************************
C     SUBROUTINE TO PRINT TIME HISTORY OF CURRENT STATE,
C     ELEMENT TYPE = 11 THREE DIMENSIONAL TENDON ELEMENTS.
C     ***************************************************
C
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      COMMON /TAPES / NIU,NOU,NT1,NT2,NT3,NT4,NT5,NT6,NT7,NT8,
     *                NT9,NT10,NT11,NT12,NT13,NT14,NT15,NT16
      COMMON /INFGR / NGR,NELS,MFST,IGRHED(10),NINFC,LSTAT,
     *                LSTF,LSTC,NDOF,DKO,DKT,EPROP(5,200)
      COMMON /INFEL / IMEM,KST,LM(18),NODE(6),INS(6),INSO(6),
     *        XX(6),YY(6),ZZ(6),PROP(5),AREA,XL(5),B(7),SEL(5),
     *        STOT(5),TT(3,3),T,TI(5),QL(12),SHA1,SHA2,
     *        BK(4,4),SK(5),IP,IDUM,ANSL,SKP(324)
C
      IF (IMEM.EQ.MFST) KHED = 0
      IF (KPR.EQ.0) RETURN
C
      IF (KHED.NE.0) GO TO 100
C
      KHED = 1
      KKPR = IABS(KPR)
      WRITE(NOU,2000) KKPR,TIME,IGRHED
100   WRITE(NOU,2100) IMEM,NODE(1),NODE(6)
      DO 200 I=1,5
200   WRITE(NOU,2010) NODE(I),NODE(I+1),SEL(I),STOT(I)
      WRITE(NOU,2200) INS
2200  FORMAT(5X,13HSLIDING INDEX,5X,6I2)
C
2000  FORMAT (///18H RESULTS FOR GROUP,I3,
     *  44H (THREE DIMENSIONAL TENDON ELEMENTS), TIME = E11.4//
     *  5X,10A4//)
2100  FORMAT (5X,8HELEM NO.,I5,5X,5HNODE1,I5,5X,5HNODE6,I5//
     *        5X,4HNODE,1X,3H---,1X,4HNODE,5X,10HTEN. FORCE,
     *        5X,10HTEN.STRESS/)
2010  FORMAT (2I9,2E15.4)
      RETURN
      END
```

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BIOGRAPHICAL SKETCH

Yuh-Yi Lin, born in Taiwan, R.O.C., on Aug. 7, 1957, received his Bachelor of Science degree in civil engineering from Tamkang University, Taiwan, in 1979. After two years of military service, he worked for the government of Chiayi, Taiwan, as a civil engineer for two years. During that period, he was involved in the design of hydraulic structures. He passed his P.E. exam to become a registered civil engineer in Taiwan in 1983. The same year, he enrolled as a graduate student at Tamkang University and received his Master of Science degree in structural engineering in 1985. He then worked for Hin-Hwa consulting firm as a structural engineer designing high-rise buildings and highway and railroad bridges. He became a registered structural engineer in 1985. In order to satisfy a strong desire to futher his education, he came to the United States to pursue his doctorate at the University of Florida in 1987. He expects to finish his Ph.D. during the spring of 1990.

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and it is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

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